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**DECISION THEORY PROCESS
FOR MAKING A MITIGATION DECISION ON
HARMONIC RESONANCE IN
POWER SYSTEMS**

**BY
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**THESIS PRESENTED IN FULFILMENT OF THE REQUIREMENTS FOR THE
DOCTOR OF PHILOSOPHY DEGREE IN ELECTRICAL ENGINEERING AT
THE UNIVERSITY OF CAPE TOWN**

SUPERVISOR: PROF. K.A. FOLLY

JULY 2003

DECLARATION

I hereby declare that the work for this thesis was done by me, that I wrote it myself and that it has not been submitted to any other educational institution in order to obtain a qualification.

Signed by candidate

G. ATKINSON-HOPE

JULY 2003

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ABSTRACT

Decision analysis is a scientific tool that is traditionally applied in business and not to electrical engineering decisions.

The reason for this research is to show how to use decision analysis to make a decision on the size of a power factor correction capacitor to be installed in an end-user plant in an electrical power system, that has the potential for harmonic resonance. How to make a decision as to whether or not mitigation is needed is also researched.

The two-stage decision theory process, developed by management scientists, to assist decision makers on making a decision when uncertainty, risk and certainty situations exist, is reviewed in this thesis. To understand the application of decision theory, worked examples are included to improve understanding and to provide a foundation for the new work introduced.

The addition of capacitors to a harmonic carrying system can result in resonance. Harmonic levels can be magnified well above accepted limits and this can cause damage to system components, especially capacitors. Recognizing and correcting a harmonic resonance problem before disastrous consequences arise is essential for system designers. Traditionally, when considering harmonic resonance, power factor correction capacitors are sized heuristically and a power factor of 0.95 is taken as a starting point. Usually, a harmonic analysis software package is used and a frequency scan study is conducted to generate a resonance curve. Resonant points are then compared to the harmonics in the system. If there is coincidence, the technique of de-tuning is applied to overcome overlapping and to choose the capacitor size. For utilities to maintain system efficiencies at acceptable levels, they encourage end-users to use a capacitor size so that the power factor has a value greater than 0.9 and as a rule of thumb, correction is not done to unity. This traditional technique is subjective and lacks decision structure.

A new three-stage decision theory process for making a harmonic resonance mitigation decision in an end-user plant is developed. Two new indices are developed to assist in making the decision. The first index assesses the severity of resonance and the second is used to make a mitigation decision. In Stage 1, a quantitative model is developed to structure and represent the decision problem with the harmonic resonance severity index as the objective function. The model uses a fixed capacitor based on full load rating as this represents the worst case. In Stage 2, Utility Theory is used as the decision criterion to select the most desirable capacitor size. In Stage 3, the mitigation index is applied to assess if mitigation is needed or not for the chosen capacitor.

Three case studies, based on deterministic models are conducted and they demonstrate the effectiveness of this newly developed decision theory process.

TABLE OF CONTENTS

	PAGE
CHAPTER 1 – INTRODUCTION	1
1.1 Background	1
1.2 Traditional approach for investigating harmonic resonance in a power system	1
1.3 Shortcomings of traditional approach	2
1.4 Need for research	6
1.5 Research boundaries	6
1.6 Main contributions of the thesis	6
1.7 Outline of the thesis	8
 CHAPTER 2 – PRINCIPLES OF RESONANCE AND HARMONIC ANALYSIS	 9
2.1 Frequency dependency of network	9
2.2 Series resonance	9
2.3 Parallel resonance	11
2.4 Harmonic analysis	13
2.4.1 Periodic waveforms	13
2.4.2 Harmonic penetration studies	14
2.4.3 Total harmonic distortion	15
2.4.4 Power and power factor	16
2.4.5 Sizing capacitors for pf correction	18
2.4.6 Harmonic sources	18
2.4.7 Computer studies on harmonic penetration	19
2.4.8 Harmonic impedance scan studies	20
2.4.9 Harmonic analysis software package	20
2.4.10 Harmonic resonance	21
2.4.11 Harmonic effects on power system devices	21
2.5 Summary	21
 CHAPTER 3 – DECISION ANALYSIS	 22
3.1 Background	22
3.2 Generalized framework	22
3.3 Need for a model	23
3.4 General structure of model	23
3.5 Certainty, uncertainty and risk classification of decision problems	24
3.6 General guidelines to modeling	24
3.7 Classification of a model	25

	PAGE
3.8 Decision theory process	26
3.9 Model building stage	27
3.9.1 Decision Table	28
3.10 Decision making stage	29
3.11 Decision making under certainty situations	31
3.12 Decision making under uncertainty situations	31
3.13 Decision making under risk situations	32
3.13.1 Meaning of risk	32
3.13.2 Approaches to probability	32
3.13.3 Classical approach	33
3.13.4 Relative frequency approach	33
3.13.5 Subjective probability	33
3.14 Probability distributions	34
3.15 Discrete probability distributions	35
3.15.1 Objective assessment method	36
3.15.1.1 Example 3.1 – Histogram for electric lamp orders	36
3.15.2 Subjective assessment method	37
3.15.2.1 Example 3.2 - Discrete probability distribution for an electrical plant	37
3.16 Continuous probability distributions	39
3.16.1 Example 3.3 – Objective assessment method	39
3.16.2 Example 3.4 – Subjective assessment method	42
3.17 Decision table for making a decision under risk	45
3.17.1 Decision making using the expected monetary rule	45
3.17.2 Example 3.5 – Application of the EMV rule	46
3.17.3 Limitations of the EMV rule	48
3.18 Utility as an evaluation model	49
3.18.1 Definition of the term utility	49
3.18.2 Utility table	50
3.18.3 Expected utility value	50
3.18.4 Axiomatic theory of utility	51
3.18.5 Variable probability method for determining utility values	52
3.18.6 Example 3.6 – Purchasing an electrical plant	53

	PAGE
3.19 Utility values for non-monetary values	56
3.20 Practical usefulness of utility theory	56
3.21 Summary	57
 CHAPTER 4 – DECISION THEORY PROCESS FOR MAKING A MITIGATION DECISION ON HARMONIC RESONANCE	 58
4.1 Background	58
4.2 Two-stage decision theory process	59
4.3 Three-stage process for making a mitigation decision on harmonic resonance	59
4.4 Define the problem and objective (block A)	61
4.4.1 Decision scenario	61
4.4.2 Problem (block A)	62
4.4.3 Objectives (block A)	62
4.5 Identification of states of nature (block B)	62
4.5.1 Recommended guideline	63
4.6 Identification of decision alternatives (block C)	64
4.6.1 Characteristics of decision alternatives	64
4.6.2 Resonance frequency band approach and the 2 – controllable input decision model	65
4.7 Objective function (block D)	72
4.8 Decision table to structure and represent the harmonic resonance decision problem	74
4.9 Application of utility theory (block E)	75
4.10 Making a mitigation decision on harmonic resonance (block F)	77
4.11 Summary	80
 CHAPTER 5 – CASE STUDY 1 – APPLICATION OF DEVELOPED DECISION THEORY PROCESS TO A SIMPLE POWER SYSTEM	 81
5.1 One-line-diagram for case study 1	81
5.2 Decision scenario	82
5.3 Stage 1 of case 1– quantitative decision model	82
5.3.1 Define the problem (block A)	82
5.3.2 Identify the objectives (block A)	82
5.3.3 Identify the states of nature (block B)	83
5.3.4 Identify decision alternatives (block C)	85

	PAGE
5.3.5 Decision table and objective function to structure and represent the harmonic resonance decision problem (block D)	99
5.4 Stage 2 – Decision making: decision 1 (block E)	102
5.5 Stage 3 – Making a mitigation decision on harmonic resonance: decision 2 (block F)	106
5.6 Summary	107
 CHAPTER 6 – CASE STUDY 2 – APPLICATION OF DEVELOPED DECISION THEORY PROCESS TO A POWER SYSTEM HAVING MULTIPLE RESONANT POINTS	 108
6.1 Description of case 2	108
6.1.1 Establishing parameter values for computer models	109
6.2 Decision theory process	110
6.2.1 Decision scenario	111
6.3 Define the problem (block A)	111
6.4 Identify the objectives (block A)	112
6.5 Identification of states of nature (block B)	112
6.6 Identification of decision alternatives (block C)	113
6.7 Objective function and calculation of outcomes (block D)	120
6.8 Develop the decision table (block D)	121
6.9 Utility theory (block E)	122
6.10 Making a harmonic resonance mitigation decision (block F)	123
6.11 Other mitigation concerns	124
6.11.1 IEEE 519 standard	124
6.11.2 Violation of mitigation criterion (%MI)	125
6.11.3 IEC61000-2-4 standard	125
6.11.4 VTHD% at PCC	125
6.11.5 Voltage levels of other capacitors	126
6.11.6 Transformer energisation	126
6.11.7 Background harmonics	126
6.12 Summary	127
 CHAPTER 7 – CASE STUDY 3 – INVESTIGATION OF THE EFFECT OF DAMPING ON THE SEVERITY OF RESONANCE AND ON THE MITIGATION DECISION IN A POWER SYSTEM HAVING MULTIPLE RESONANT POINTS	 129
7.1 Background	129
7.2 Decision scenario	129
7.3 Define the problem (block A)	130

	PAGE
7.4 Objectives (block A)	130
7.5 States of nature (block B)	130
7.6 Decision alternatives (block C)	130
7.7 Relevant decision table (block D)	131
7.8 Utility table (block E)	131
7.9 Decision 2 (block F)	132
7.10 VTHD% at PCC	133
7.11 Mitigation of harmonics	133
7.12 Mitigation solutions for case 3	133
7.13 Mitigation solution 1	134
7.14 Mitigation solution 2	137
7.15 Mitigation solution 3	140
7.16 Recommended mitigation solution	142
7.17 Summary	143
CHAPTER 8 – CLOSURE	144
8.1 Contributions	144
8.2 Conclusions	145
8.3 Recommendations	147
8.4 Future work	148
REFERENCES	149
APPENDICES	153
Appendix 1 - International Standards (limits) for PF Correction Capacitors	153
Appendix 2 - Computer Model for SCAN Study – Case 1	159
Appendix 3 - Computer Model for Harmonic Penetration Study – Case 1	161
Appendix 4 - Calculation of Outcome Values for Decision Table – Case 1	163
Appendix 5 – Case 1-Harmonic Penetration Results, calculation of mitigation indices and comparison to international capacitor loading standards	167
Appendix 6 - Computer Models, derivation of utility function and mitigation indices – Case 2	172
Appendix 7 – Calculation of mitigation indices-Harmonic Filter Theory and Computer Model including 2 nd – Order Damped Filter at PCC for Case 3	187
Appendix 8 - Computer Model including Notch Filter at Bus6B – Case 3	198
Appendix 9 – Examples – Decision analysis	202

LIST OF TABLES

TABLES	PAGE
3.1 Decision table	29
3.2 Comparison of criteria for decision making under uncertainty	31
3.3 Frequency distribution for electrical lamps	36
3.4 Frequency distribution for temperature at an electrical plant	40
3.5 Decision table for making a decision under risk	45
3.6 Decision table – new electrical plant	47
3.7 Relevant decision table – new electrical plant	48
3.8 Utility table for making a decision under risk	50
3.9 Decision table for purchasing an electrical plant	53
3.10 Utility table for purchasing an electrical plant	54
4.1 Controllable input 2, alternatives per %Q _C size	70
4.2 Decision alternatives 2 – Controllable inputs	72
4.3 %HRSI qualitative categories	74
4.4 Severity of harmonic resonance decision table	74
4.5 Utility table for making a decision under risk	75
4.6 Elicitation session	76
4.7 Mitigation decision 1	77
4.8 Mitigation decision 2	78
5.1 Resonant point co-ordinates	87
5.2 Impedance values	89
5.3 %Q _C calculations	90
5.4 Choice value for %Q _C	95
5.5 Constrained controllable input 1	97
5.6 a ₁ curve	98
5.7 a ₂ curve	98
5.8 Identification of decision alternatives for Case 1	99
5.9 Severity of harmonic resonance decision table	99
5.10 Decision table for Case 1	101
5.11 Relevant decision table for Case 1	101
5.12 Utility table for Case 1 (risk averse)	105
5.13 Mitigation decision 1 for Case 1	105
5.14 Mitigation decision 2 for Case 1	106

TABLES	PAGE
6.1 Capacitor bank step setting sizes	114
6.2 Co-ordinates of resonance points for step settings	115
6.3 Capacitor sizes in range 1	116
6.4 Co-ordinates for resonance of points of constrained RFB	117
6.5 The 2 – controllable inputs, alternatives	120
6.6 Severity of harmonic resonance decision table – Case 2	121
6.7 Relevant decision table – Case 2	122
6.8 Utility table (risk averse) – Case 2	122
6.9 Mitigation decision 1 – Case 2	123
6.10 Harmonic penetration results	123
6.11 Mitigation decision 2 – Case 2	123
7.1 Relevant decision table – Case 3	131
7.2 Utility table – Case 3	131
7.3 Mitigation decision 1 – Case 3	132
7.4 Mitigation decision 2 – Case 3	132
7.5 2 nd Order harmonic filter results-mitigation solution 1	137
7.6 Notch filter results-mitigation solution 2	139
7.7 Notch filter results-mitigation solution 3	142

LIST OF FIGURES

FIGURE	PAGE	
1.1	Equivalent diagram under linear load conditions	3
1.2	Equivalent diagram under non-linear load conditions	4
3.1	General structure of a decision model	24
3.2	Decision theory process	26
3.3	Decision situations	30
3.4	Subjective probability assessment	34
3.5	Probability function (histogram)	37
3.6	Frequency distribution	38
3.7	Discrete probability distribution	39
3.8	Histogram for temperature	40
3.9	Histogram with narrow class intervals	40
3.10	Probability density function	41
3.11	Cumulative distribution function	41
3.12	Histogram for electrical demand	43
3.13	Probability density function for 2000kVA plant	44
3.14	Cumulative distribution function for 2000kVA plant	44
3.15	Utility function – decision-maker electrical plants	55
4.1	Decision theory process for making a mitigation decision on harmonic resonance	60
4.2	System with a potential for severity of harmonic resonance	61
4.3	Equivalent circuit	61
4.4	Power triangle for end-user plant	65
4.5	Power triangle with capacitor installed	66
4.6	System frequency response as capacitor size is varied	66
4.7	Template for making a mitigation decision on harmonic resonance	79
5.1	One-line diagram – Case 1	81
5.2	Frequency distribution (S_E)	84
5.3	Discrete probability distribution (S_E)	85
5.4	Resonance frequency band	86
5.5	Power triangle	90
5.6	Characteristic harmonic resonance frequency band	91
5.7	Characteristic harmonics within range 2	92
5.8	Sub-regions A, B and C within range 2	94

FIGURE	PAGE
5.9 Controllable inputs 1	96
5.10 Constrained controllable inputs 1 and 2	97
5.11 Utility function for Case 1	104
6.1 Description of Case 2	108
6.2 Resonance close to or at 5 th harmonic	110
6.3 Resonance frequency band – Case 2 (bus 5AA)	114
6.4 Range 1 – Case 2	116
6.5 RFB and CHRFB – Case 2	119
6.6 Identified controllable input 1, alternatives	123
7.1 Resonance curves $X/R = 24$ – Case 3	130
7.2 SCAN results at PCC	136
7.3 SCAN results at bus 5AA	136
7.4 SCAN results notch filter at bus 6B	139
7.5 SCAN results notch filter at bus 5AA	141

ABBREVIATIONS AND SYMBOLS

a_0	dc component	FL	full load
A	event(state of nature)	f_s	series resonant frequency
a_n	set of decision alternatives	f_p	parallel resonant frequency
\sum	algebraic sum	f_{lowest}	lowest frequency
(A)	outcome	G	conductance
a_{nm}	hch	$>$	greater than
B	susceptance	h_R	harmonic resonance frequency
B	buy	h	harmonic, multiple of f_1
b^*	optimal purchase quantity	hch	characteristic harmonic
B	event	h^-	negative sequence component
(B)	outcome	h^+	positive sequence component
C	capacitance	HRSI	harmonic resonance severity index
c	cost	h_{notch}	notch frequency
cdf	cumulative distribution function	Hpu	frequency in per-unit (h/f_1)
(C)	outcome	I_{sc}	short-circuit current
C	event	I_h	harmonic current
CHRFB	characteristic harmonic resonance frequency band	I_m	imaginary component of current
CAP	capacitor	I	current
C_f	filter capacitor	I_c	capacitor current
D	distortion voltamperes	I_L	inductor current
dpf	displacement power factor	I_x	phasor sum of currents
d	demand	I_{RMS}	root mean square value of current
\$	dollar	i_a	current in phase "a"
Δ	change (delta)	I_d	dc component of current
Φ	empty set	$I_{C(RMS)}$	root mean square value of current of a complex wave
E	event	I_1	rated current
E(X)	expected value	IEEE	Institute of Electrical Electronic Engineers
EU	expected utility value	IEC	International Electrotechnical Commission
EMV	expected monetary value	I_{source}	current source
fr	resonant frequency	I^2R	copper losses
f_1	nominal system frequency	kg	kilo-grams
f	frequency	kV	kilo-volts
$f(t)$	fourier series	kA	kilo-amperes
$f(x)$	probability density function	kvars	kilo-voltamperes
$F(x)$	cumulative distribution function		

L_s	source inductance	$R(\omega)$	alternating current resistance
L	inductance	R_e	real component
L_f	filter inductance	R_w	winding resistance
\leq	less than or equal to	r_{nm}	set of result variables
$<$	less than	RFB	resonance frequency band
		R_R	reactor resistance
MI	mitigation index		
mpt	midpoint	S_{sys}	system fault level
MVA	Mega-voltamperes	SCC	short-circuit capacity
		S_b	base apparent power
n^{th}	harmonic component	S	apparent power
N	non-active power	S^*	optimal selling price
\cap	no common point between A and B	s	selling price
\neq	not equal to	S_N	set of states of nature
		s_d	electrical demand
		S_E	end-user apparent power
OBF	objective function		
Ω	ohms	θ	phase angle
ω	angular frequency	$\theta(\omega)$	phase angle as a function of ω
ω_1	fundamental angular frequency	t	time
ω_R	angular frequency at resonance	T_o	period of fundamental wave
		THD	total harmonic distortion
		T	transformer
pu	per-unit	TOP	output processor program
P	real power		
pf	power factor	$U(V)$	expected utility value
P_h	harmonic power	U_{nN}	utility measure
P^*	optimal profit	U	utility
p	probability	U_N	nominal voltage
p_n	set of probability variables		
pdf	probability density function	V_s	source voltage
p_{ind}	probability indifference value	V_n	voltage to neutral
%	percentage	V_b	base voltage
Φ	phase angle	V_x	phasor sum of voltages
PCC	point of common coupling	V_L	voltage across inductor
		V_C	voltage across capacitor
Q_C	capacitor voltamperes reactive	V	volts
Q_{ser}	quality factor series circuit	$ V $	absolute value of voltage
Q_{par}	quality factor parallel circuit	V_{RMS}	root mean square value of voltage
Q	reactive power	v	salvage value
Q_h	reactive power at h	V_D	driving point voltage
Q_1	size of capacitor at f_1	V_{C1}	rated capacitor voltage
$\%Q_C$	capacitor size(%)		
		X_{source}	source reactance
R	resistance	X_c	capacitor reactance

$X(\omega)$	imaginary component
X_L	inductive reactance
X_t	transformer reactance
X	reactance
x	fraction for shifting frequency below f_{lowest}
X_R	filter reactor reactance
Y	admittance
Z_b	base impedance
Z	impedance
Z_L	pure inductive component
Z_C	pure capacitive component
Z_R	pure resistive component
$ Z $	absolute value of impedance
$Z(\omega)$	impedance as a function of ω
Z_h	harmonic impedance
Z_D	driving point impedance
Z_F	filter impedance

LIST OF APPENDICES

1. International Standards (limits) for PF Correction Capacitors.
2. Computer Model for SCAN Study – Case 1.
3. Computer Model for Harmonic Penetration Study – Case 1.
4. Calculation of Outcome Values for Decision Table – Case 1.
5. Case 1-Harmonic Penetration Results, calculation of mitigation indices and comparison to international capacitor loading standards.
6. Computer Models, derivation of utility function and mitigation indices – Case 2.
7. Calculation of mitigation indices - harmonic filter theory and Computer model including 2nd – order damped filter at PCC for Case 3.
8. Computer Model including Notch Filter at Bus6B – Case 3.
9. Examples - Decision analysis.

CHAPTER 1

INTRODUCTION

1.1 BACKGROUND

Non-linear loads, such as adjustable speed drives inject harmonic currents into power systems and are rapidly replacing linear-loads in end-user plants causing distortion.

Power factor is a measure of the efficiency of a system and indicates the utilization of the system's capacity. The closer the power factor is to unity the more efficient the system is being operated. When power factors are low (e.g., 0.7) utilities generally encourage customers to improve them to a minimum of 0.9 so that wasted system capacity is reduced. Shunt capacitor banks for power factor correction are applied to end-user plants to improve power factors and increase overall power system efficiencies. Typically, an acceptable limit to aim for is a power factor of 0.95. As a rule of thumb, correction to unity power factor is not done in industry [1], [2], [32], [46], [47].

Capacitive reactance is inversely proportional to frequency. Therefore capacitors in systems present a low reactance path to harmonics and this can lead to an over-current causing capacitor failure. Even worse, the addition of capacitors to a harmonic carrying system can result in a resonant tuned circuit, causing specific levels of voltage and/or current to be magnified well above accepted limits. A consequence of this is that capacitors in a system could be damaged and/or destroyed [2].

System designers are interested to know if the capacitor installed would cause harmonic resonance and what is the extent of the problem. Recognizing and correcting the resonance problem before it can cause a disastrous consequence is important. How this problem is traditionally approached is now reviewed. The shortcomings of this approach are deduced and a new process for investigating this problem is introduced in this thesis. [3], [7] .

1.2 TRADITIONAL APPROACH FOR INVESTIGATING HARMONIC RESONANCE IN A POWER SYSTEM

The traditional approach used in power systems to investigate a potential harmonic resonance problem (although not clearly defined in literature) is as follows [47]:

1) Firstly, the resonant frequency (f_r) is predicted from the ratio of the fault level to the capacitor size [2], [3], [4], [5], namely:

$$\frac{f_r}{f_1} = \sqrt{\frac{S_{sys}}{Q_c}} \quad (1.1)$$

where: f_r = resonant frequency in hertz
 f_1 = nominal system frequency in hertz
 S_{sys} = system fault level in voltamperes
 Q_C = voltamperes reactive for capacitor

For an end-user plant having a low power factor, a value of 0.95 is selected as a start point for the investigation. A capacitor size " Q_C " to improve the power factor to 0.95 and the fault level (S_{sys}) are determined. Using equation 1.1, the resonant frequency (f_r) is predicted. This step is sometimes simplified by substituting for " S_{sys} ", the reactance for the transformer " X_t " supplying the end-user plant as it is assumed to provide the bulk of the impedance under short circuit conditions.

2) Using the value " Q_C " to give a 0.95 power factor improvement, " f_r " is calculated. If the calculated value is near a characteristic harmonic (hch), then by trial and error different " Q_C " values are applied until " f_r " is not near any " hch ".

3) A harmonic analysis software package is then used to conduct a frequency scan study for the determined " Q_C " value. A resonance curve is generated to check that any " f_r " do not coincide with a " hch ".

4) The harmonic analysis package is then used to calculate the voltage " V_C " across the capacitor as well as the capacitor current " I_C ". These values are then checked against IEEE standards for capacitors (Appendix 1). If the values exceed any of the standards then mitigation (Appendix 7) in the form of a filter is applied to prevent damage being caused to the installed capacitor.

This heuristic approach is generally successful and the average engineer could accomplish the given task.

1.3 SHORTCOMINGS OF TRADITIONAL APPROACH

To understand the limitations behind the traditional approach the following explanations are given.

(i) Under sinusoidal and linear conditions and with resistance neglected, consider a power system represented by its Thevenin equivalent diagram (V_s , X_{source}) as shown in Figure 1.1 [5].

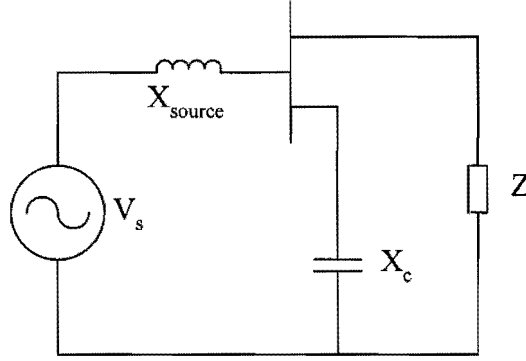


Figure 1.1 Equivalent diagram under linear load conditions

Let “SCC” and “ I_{sc} ” be the short circuit capacity and current, respectively.

If $X_{source} = 2\pi f_1 L_s$ and $X_c = 1/2\pi f_1 C$, then with a short circuit on V_n ,

$$I_{sc} = V_s / X_{source} \quad (1.2)$$

and, $SCC = V_s I_{sc} \quad (1.3)$

the short circuit reactance is:

$$X_{source} = V_s^2 / SCC \quad (1.4)$$

Working in per-unit (pu), then:

$$X_{sourcepu} = \frac{X_{source}}{Z_b} = \frac{V_s^2 / SCC}{V_b^2 / S_b} = \frac{V_s^2 S_b}{V_b^2 SCC} \quad (1.5)$$

If “ S_b ” and “ V_b ” are the base apparent power and voltage, respectively and “ V_b ” is assumed to be equal to “ V_s ” and “ V_c ” is the voltage across the capacitor, then,

$$X_{cpu} = \frac{X_c}{V_b^2} \frac{Q_c}{Q_{Cpu}} = \frac{X_c}{V_b^2} \frac{V_c^2}{X_c} \frac{1}{Q_{Cpu}} \quad (1.6)$$

but, $V_b = V_c$

$$X_{cpu} = \frac{1}{Q_{Cpu}} \quad (1.7)$$

(ii) To investigate the system when harmonics are present, let the linear load “Z” in figure 1.1 be replaced by a non-linear load so that harmonics can be injected into the system. Let this replacement be represented by a harmonic current source (I_h) and let the sinusoidal voltage source V_s be short-circuited. Figure 1.2 shows the equivalent diagram ($X_{source} // X_c$):

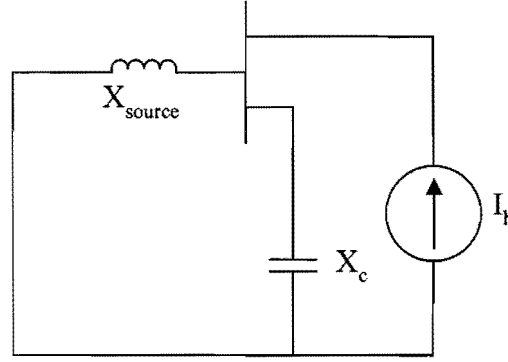


Figure 1.2 Equivalent diagram under non-linear load conditions

The system resonant frequency is:

$$\begin{aligned}
 fr &= \frac{1}{2\pi\sqrt{L_s C}} = \frac{1}{2\pi\sqrt{(X_{source} / 2\pi f_1)(1 / 2\pi f_1 X_c)}} = \frac{1}{\frac{2\pi}{2\pi f_1} \sqrt{\frac{X_{source}}{X_c}}} \\
 &= \frac{f_1}{\sqrt{\frac{X_{source}}{X_c}}} = f_1 \sqrt{\frac{X_c}{X_{source}}} = f_1 \sqrt{\frac{X_{cpu} Z_b}{X_{sourcecpu} Z_b}} = f_1 \sqrt{\frac{X_{cpu}}{X_{sourcecpu}}} \\
 &= f_1 \sqrt{\frac{\frac{1}{Q_{Cpu}}}{\frac{1}{SCC_{pu}}}} = f_1 \sqrt{\frac{SCC_{pu}}{Q_{Cpu}}} = f_1 \sqrt{\frac{SCC/S_b}{Q_c/S_b}}
 \end{aligned}$$

As $SCC = S_{sys}$ (system fault level), therefore:

$$\frac{fr}{f_1} = \sqrt{\frac{S_{sys}}{Q_c}} \quad (1.8)$$

Thus equation (1.8) equals equation (1.1). If we introduce the resonance order “hr”, defined as $hr = fr/f_1$, then:

$$hr = \sqrt{\frac{S_{sys}}{Q_c}} \quad (1.9)$$

Literature has the following to say about this approach:

- a) The equation (1.9) is only good for one of each kind of element in a system. It is inaccurate as no upstream capacitors are taken into account [4]. The approach does not give the correct resonant harmonic frequencies for systems containing a complex interconnection of many capacitances and inductances. The approach (equation 1.9) is too crude to be practically useful as it is too simple. It also only predicts one resonant frequency and is therefore invalid for real systems wherein multiple resonant points exist.
- b) It is stated in [4] that if there is an existence of harmonic resonance, this does not imply that a problem would occur since a system may provide a high value of damping which can have a significant impact on the severity of the resonance. Thus it is not always necessary to provide mitigation despite the capacitor resonating with the system at key harmonic frequencies.
- c) Reference [3] states that a more accurate method for assessing system capacitor resonance conditions than the equation (1.9) approach is required. It is necessary to determine if a harmonic resonance condition exists and important to evaluate if the resonance is “*severe*” or not and to “*decide*” if preventive action (mitigation) needs to be taken. This reference proposes that a measurement method be used to improve on the equation (1.9) approach.

In summary, the approach used in equation (1.9) has the following shortcomings:

- a. It does not reveal the extent of the problem and does not assess the severity of harmonic resonance [3].
- b. Even though the approach can detect the existence of a harmonic resonance, this does not imply that damage to the capacitor will incur [3].
- c. The results of the approach can be misleading, in that, even if the approach indicates resonance at a key harmonic frequency it is not always necessary to mitigate the resonance [3].
- d. The approach will not give the correct harmonic resonant frequencies for systems containing upstream capacitances and inductances [4].

The selection of the capacitor size to be used in equation (1.9) is heuristically determined. The problem with this traditional technique is that it is subjective and lacks a definite decision structure. How to make a “*decision*” on the size of capacitor to be installed in terms of the severity of harmonic resonance is not disclosed in literature. How to decide if preventative action (mitigation) is needed or not to protect the capacitor in terms of severity of harmonic resonance is also not disclosed.

The measurement procedure proposed in reference [3], to improve on the fault/capacitor method does not quantify the severity of harmonic resonance and the procedure is limited to a real system in which the capacitor has already been installed. The reference also does not disclose a computer method for the investigation of severity of harmonic resonance and choosing a capacitor size.

1.4 NEED FOR RESEARCH

In view of the shortcomings found it can be said that there is a need for:

- a. A more accurate method for assessing system capacitor resonance conditions than that given by the equation (1.9) approach.
- b. The assessment of the severity of the resonance condition in addition to the resonance frequency.
- c. A “*decision making method*” (in terms of severity of harmonic resonance) to decide:
 - (1) On the size of the capacitor to be installed.
 - (2) Whether or not preventative action (mitigation) is needed to protect the capacitor from damage.

1.5 RESEARCH BOUNDARIES

This research is conducted within the following boundaries:

A fixed capacitor based on full load conditions is used in the development of the new approach and the subsequent investigations as it represents the worst case. Thus, capacitor switching as the load varies is not considered in this thesis. The capacitor size chosen in this thesis is constrained to a power factor value greater than 0.9 and correction to unity is not considered. How to formulate and analyze the problem and make an informed decision on the selection of the capacitor is the focus for this research. The thesis is limited to decision analysis to solve the problem.

1.6 MAIN CONTRIBUTIONS OF THE THESIS

A new approach to analyzing and making a decision on harmonic resonance in power systems is developed. A new structured decision making process based on the scientific method of decision analysis is developed to help make a decision on the severity of harmonic resonance and to decide between different sizes of capacitors. In addition, the problem of whether or not mitigation is needed is also addressed.

The main contributions of the thesis are:

- A new three-stage decision theory process is developed. In stage 1, a new quantitative model is developed for analyzing harmonic resonance. In stage 2, a new application of a scientific theory is introduced to choose the size of capacitor to be installed. In stage 3, a new step is introduced for making a decision about preventative action and is called the mitigation decision. A new index, called the mitigation index (MI) is introduced to help make the mitigation decision and to quantify the level at which mitigation is needed under the severity of harmonic resonance conditions.
- The quantitative model is developed so that a decision on the severity of harmonic resonance can be made. A new objective function, called the Harmonic Resonance Severity Index (HRSI) is introduced to quantify the outcome of the model in terms of the level of severity at key harmonic frequencies and takes into account the model's major conceptual ingredients such as decision alternatives (controllable inputs) and states of nature (uncontrollable inputs).
- A new application of a decision table is developed to represent the model where the outcome is the objective function. A new 2-Controllable Input approach is developed as opposed to the traditional 1-Controllable Input approach generally used in business applications of decision analysis. The first of the two controllable inputs comprises different sizes of capacitors to choose from, while the second of the two inputs are key harmonics injected into the system. The states of nature introduced to the model represent loading conditions of the end-user plant in which the capacitor is to be installed and probabilities are assigned so that each state of nature has a likelihood of occurrence.
- A new application of Utility theory is introduced for making a decision on the size of capacitor to be installed. Furthermore, a Variable Probability method for calculating utility values for a utility table is introduced. An Expected (Value) Utility equation, quantified in terms of the severity of harmonic resonance is introduced to assist in choosing the decision alternative which best meets the objective.
- Three case studies are conducted to demonstrate the effectiveness of this newly developed decision theory process.

1.7 OUTLINE OF THE THESIS

The outline of the thesis is as follows. In chapter 2, the principles of resonance and harmonic analysis are reviewed. In chapter 3, the concepts of a quantitative decision model are explained. The traditional decision process developed by management scientists is described and decision making under risk is reviewed in more detail as it is the decision zone most relevant to the topic researched. Numerous worked examples (business related) are included in chapter 3 to improve understanding and lay down a foundation for the new work that was done. In chapter 4, a new three-stage decision theory process for making a mitigation decision on harmonic resonance in power systems is developed. The general theory and principles behind this new approach are discussed.

In stage 1 of the process a quantitative decision model is developed, which includes a new objective function and decision table for conceptualizing, analyzing and solving the decision problem on the severity of harmonic resonance. In Stage 2, a new application for utility theory (severity of harmonic resonance) is introduced to select the decision alternative that best meets the objective of the decision problem. In stage 3, a new mitigation decision method is developed. It is specifically aimed at severity of harmonic resonance and whether or not mitigation (preventative action) is needed to prevent damage to a new power factor correction capacitor to be installed in a plant.

Three case studies are conducted to demonstrate the effectiveness of the developed process in chapters 5, 6 and 7, respectively. In case study 1, the developed decision theory process is applied to a simple power system. In case study 2, the developed process is applied to a power system having multiple resonant points. Case study 3 investigates the effect of damping on the “Harmonic Resonance Severity Index (HRSI)” and on the “Mitigation Index (MI)” in a power system having multiple resonant points. Mitigation concerns are discussed at the end of chapter 6. In chapter 7, possible mitigation solutions for case study 3 are discussed. In chapter 8, contributions, conclusions, recommendations and ideas for future studies are given.

CHAPTER 2

PRINCIPLES OF RESONANCE AND HARMONIC ANALYSIS

The purpose of this chapter is to review the principles of series and parallel resonance and harmonic analysis. Procedures for conducting harmonic penetration and impedance scan studies as well as the effects of harmonics on power system devices are reviewed.

2.1 FREQUENCY DEPENDENCY OF NETWORK

Impedance and its dependency on frequency plays an important role in resonance and harmonic studies.

To emphasize the complex and frequency dependant properties of impedance, they are written in the form [8]:

$$Z = Z(j\omega) = R(\omega) + j X(\omega) \quad (2.1)$$

where: $R(\omega) = R_e[Z]$ and $X(\omega) = I_m[Z]$ (2.2)

The real part $R(\omega)$ is called ac resistance. The imaginary part $X(\omega)$ is reactance.

Equation (2.1) also holds for each of the individual R, L, C, elements, since $Z_R = R = R + j0$, $Z_L = j\omega L = 0 + j\omega L$ and $Z_C = 1/j\omega C = 0 + j(-1/\omega C)$. Furthermore for an inductor or capacitor alone, $X_L = I_m[Z_L] = \omega L$, $X_C = I_m[Z_C] = -1/\omega C$.

Nevertheless, we can express Z in polar notation:

$$Z = |Z| \angle \theta \quad (2.3)$$

with,

$$|Z| = \sqrt{R^2(\omega) + X^2(\omega)} \quad \theta = \angle Z = \tan^{-1} \frac{X(\omega)}{R(\omega)} \quad (2.4)$$

Admittance (reciprocal of impedance) is also a complex and a frequency dependent quantity, hence we write it as:

$$Y = Y(j\omega) = G(\omega) + j B(\omega) \quad (2.5)$$

where:

$$G(\omega) = R_e[Y] \quad \text{and} \quad B(\omega) = I_m[Y] \quad (2.6)$$

The real part $G(\omega)$ is called ac conductance and the imaginary part $B(\omega)$ is susceptance.

2.2 SERIES RESONANCE

Inductors and capacitors have opposite properties in two respects. Inductive reactance ($X_L = \omega L$) is positive and increases with frequency, whereas capacitive reactance ($X_C =$

$-1/\omega C$) is negative and decreases with frequency. These properties lead to undesirable effects in circuits, which contain both types of reactive elements.

Depending on the excitation frequency, either the inductance or capacitance will dominate or the two reactances may cancel out and produce the phenomenon known as resonance.

Consider a series RLC network. Its terminal impedance is:

$$Z(j\omega) = R + j\omega L - j/\omega C = R + jX(\omega) \quad (2.7)$$

where,

$$X(\omega) = \omega L - 1/\omega C \quad (2.8)$$

The capacitance dominates at low frequencies and the net reactance is negative. The inductance dominates at high frequencies and the net reactance is positive. The borderline between these two cases occurs at $\omega = \omega_R$, when:

$$X(\omega_R) = \text{Im}[Z(j\omega_R)] = 0 \quad (2.9)$$

This defines the series resonance condition.

The corresponding resonant frequency for an RLC network must satisfy,

$$\omega_R L - 1/\omega_R C = 0, \text{ so } \omega_R^2 = 1/LC \text{ and,}$$

$$\omega_R = 1/\sqrt{LC} \quad (2.10)$$

thus,

$$Z(j\omega_R) = R + jX(\omega_R) = R \quad (2.11)$$

The network appears to be purely resistive at resonance. The magnitude and angle of $Z(j\omega)$ of this network is calculated from,

$$|Z(\omega)| = \sqrt{R^2 + (\omega L - 1/\omega C)^2} \quad (2.12)$$

$$\theta(\omega) = \tan^{-1} \frac{\omega L - 1/\omega C}{R} \quad (2.13)$$

This shows that $|Z(\omega)|$ has a unique minimum at ω_R , and that $\theta(\omega)$ goes from -90° to $+90^\circ$ as ω increases.

Now if V_L and V_C are the voltages across the L and C elements, respectively and we let, $V_x = V_L - V_C$, then at resonance $V_x = 0$. However, V_L and V_C may actually have large amplitudes, perhaps even larger than the amplitude of the supply voltage (V_s). Now,

$$I = V_s / Z(j\omega_R) = V_s / R, \text{ then with } \omega = \omega_R = 1/\sqrt{LC}$$

thus,

$$V_L = j\omega_R LI = \frac{jV_s}{R} \sqrt{\frac{L}{C}} \quad \frac{V_L}{V_s} = \frac{j}{R} \sqrt{\frac{L}{C}} \quad (2.14)$$

$$V_C = \frac{I}{j\omega_R C} = -\frac{jV_s}{R} \sqrt{\frac{L}{C}} \quad \frac{V_C}{V_s} = -\frac{j}{R} \sqrt{\frac{L}{C}} \quad (2.15)$$

The quality factor of the series RLC network is defined as:

$$Q_{ser} = \frac{\omega_R L}{R} = \frac{1}{R} \sqrt{\frac{L}{C}} \quad (2.16)$$

The quality factor gives an indication of the amplification of V_L and V_C with respect to V_s , thus at resonance,

$$\frac{V_L}{V_s} = jQ_{ser} \quad V_L = jQ_{ser} V_s \quad (2.17)$$

$$\frac{V_C}{V_s} = -jQ_{ser} \quad V_C = -jQ_{ser} V_s \quad (2.18)$$

If $Q_{ser} > 1$, then the amplitudes $|V_L|$ and $|V_C|$ will exceed $|V_s|$, an effect known as resonant voltage rise [8].

2.3 PARALLEL RESONANCE

If, $Z_R = R + j0$, $Z_L = 0 + j\omega L$ and $Z_C = 0 + 1/j\omega C$, the admittance of a parallel network with $Z_R // Z_L // Z_C$ is:

$$Y(j\omega) = G + j\omega C - j/\omega L = G + j B(\omega) \quad (2.19)$$

where:

$$G = \frac{1}{R} \text{ and } B(\omega) = \omega C - 1/\omega L \quad (2.20)$$

The susceptance $B(\omega)$ changes sign as frequency increases, and we define the parallel resonance condition by:

$$B(\omega_R) = \text{Im} [Y(j\omega_R)] = 0 \quad (2.21)$$

and the resonant frequency is,

$$\omega_R = 1/\sqrt{LC} \quad (2.22)$$

The parallel network also appears to be purely resistive at resonance, since $Y(j\omega_R) = G = 1/R$.

To demonstrate the difference between a series and parallel resonant circuit, substitute equation (2.19) into $Z(j\omega) = 1/Y(\omega)$ to obtain the impedance and its angle, namely:

$$|Z(\omega)| = \frac{1}{\sqrt{G^2 + (\omega C - 1/\omega L)^2}} \quad (2.23)$$

$$\theta(\omega) = -\tan^{-1} \frac{\omega C - 1/\omega L}{G} \quad (2.24)$$

If the frequency is varied and $|Z(\omega)|$ and $\theta(\omega)$ are plotted, we see that $|Z(\omega)|$ has a unique maximum at ω_R and that $\theta(\omega)$ goes from $+90^\circ$ to -90° as ω is increased. This is opposite to the series RLC network. A further difference is that in parallel resonance there is a resonant current rise as opposed to resonant voltage rise. Even though the current $I_x = I_C - I_L = 0$ when $\omega = \omega_R$, the reactive current rises are given by:

$$I_C = j Q_{\text{par}} I \quad I_L = -j Q_{\text{par}} I \quad (2.25)$$

In the parallel case, the quality factor Q_{par} is the dual of Q_{ser} and is given by:

$$Q_{\text{par}} = \frac{R}{\omega_R L} = R \sqrt{\frac{C}{L}} \quad (2.26)$$

thus, when the element values are equal,

$$Q_{\text{par}} = \frac{1}{Q_{\text{ser}}} \quad (2.27)$$

Next we need to demonstrate the role played by the winding resistance R_w associated with a real inductor ($R_w + j\omega L$). To do this a parallel circuit having a real inductor in parallel with a capacitor is used. R_w is assumed to be the only resistance in the network. The admittance of the network is then:

$$Y(j\omega) = j\omega C + \frac{1}{R_w + j\omega L} = \frac{j\omega C R_w - \omega^2 LC + 1}{R_w + j\omega L} \quad (2.28)$$

As R_w is usually small, the analysis can be simplified, that is, $R_w + j\omega L \approx j\omega L$ then:

$$Y(j\omega) = \frac{CR_w}{L} - \frac{\omega C}{j} + \frac{1}{j\omega L} \quad (2.29)$$

$$Y(j\omega) = (CR_w / L) + j(\omega C - 1/\omega L) \quad (2.30)$$

Thus, $I_m[Y(j\omega_R)] = 0$ at $\omega_R = 1/\sqrt{LC}$. The analysis reveals that R_w acts in a similar manner as a parallel network with $Z_R // Z_L // Z_C$ (equation 2.19) near ω_R , thus:

$$G = \frac{CR_w}{L} \quad \text{and} \quad R_{\text{par}} = \frac{L}{CR_w} = \frac{1}{G} \quad (2.31)$$

The formulae (equations 2.1 to 2.31) derived above are used to do calculations on series and parallel resonance.

2.4 HARMONIC ANALYSIS

2.4.1 PERIODIC WAVEFORMS

A waveform $f(t)$ is periodic if for some $T > 0$ and all t , [9]

$$f(t + T) = f(t) \quad (2.32)$$

T is the period of the waveform. The fundamental period is the smallest positive real number (T_0), thus the fundamental frequency (Hz) of the waveform is,

$$f_1 = 1/T_0 \quad (2.33)$$

and the fundamental angular frequency (in rad/sec) is given by,

$$\omega_1 = 2\pi f_1 = 2\pi/T_0 \quad (2.34)$$

Periodic waveforms found in power systems have a decomposition as the sum of sinusoidal functions:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos(n\omega_1 t) + b_n \sin(n\omega_1 t)) \quad (2.35)$$

The sine and cosine terms can be combined to give an equivalent decomposition:

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_1 t + \theta_n) \quad (2.36)$$

Both equations (2.35) and (2.36) are called the trigonometric Fourier series representation of $f(t)$.

The first term, $a_1 \cos(\omega_1 t + \theta_1)$ is called the fundamental component of $f(t)$ as has a magnitude a_1 and phase angle θ_1 . The second term of the expansion, $a_2 \cos(2\omega_1 t + \theta_2)$ is called the second harmonic of $f(t)$ with magnitude a_2 and phase angle θ_2 . Similarly for the remaining terms $a_n \cos(n\omega_1 t + \theta_n)$.

Harmonics are therefore multiples of the fundamental frequency.

Periodic functions which contain no dc component (a_0) nor even harmonics form a class said to have half wave symmetry and have the property that [10]

$$f(t) = -f(t + T/2) \quad (2.37)$$

This means that the period can be divided into two consecutive halves, one of which is the reflection of the other in the time axis.

This class of periodic waveform is commonly found in power systems under steady-state conditions. This means that the current flowing through components see the same characteristic in either the positive or negative direction.

2.4.2 HARMONIC PENETRATION STUDIES

Relative to circuit analysis, equation (2.36) has the advantage that it expresses periodic functions directly in terms of ac components ($a_0 = 0$).

If a network is driven by a periodic excitation $f(t)$, each ac input component (each harmonic) produces an output component at the same frequency.

A further advantage of using a Fourier series to represent a distorted waveform is that it is easier to find the systems response to an input that is sinusoidal and the conventional steady-state analysis techniques can be used. The system can then be analyzed separately at each harmonic. The outputs at each harmonic are then combined to form a new series and a new output waveform can be determined [11].

The computation of harmonic currents and voltages throughout an ac system in the presence of one or more current harmonic sources is called “harmonic penetration”. When harmonic currents are injected in a multi-port system and if it is assumed that the system is linear and passive, the principle of superposition may be applied to enable each to be considered independently. The harmonic current penetrates into the system and reacts with the system to cause harmonic voltages to appear. The resultant harmonic voltages are calculated using nodal analysis. The nodal admittance $[Y_h]$ of the system at a frequency “h” is given by [12], [13]:

$$[Y_h] = \begin{bmatrix} Y_{11} & Y_{12} & \dots & Y_{1i} & \dots & Y_{1k} & \dots & Y_{1n} \\ Y_{21} & Y_{22} & \dots & Y_{2i} & \dots & Y_{2k} & \dots & Y_{2n} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ Y_{i1} & Y_{i2} & \dots & Y_{ii} & \dots & Y_{ik} & \dots & Y_{in} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ Y_{k1} & Y_{k2} & \dots & Y_{ki} & \dots & Y_{kk} & \dots & Y_{kn} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ Y_{n1} & Y_{n2} & \dots & Y_{ni} & \dots & Y_{nk} & \dots & Y_{nn} \end{bmatrix} \quad (2.38)$$

where: Y_{ki} : mutual admittance between busbars k and i at “h”.
 Y_{ii} : self admittance busbar i at “h”.

The system harmonic voltages are calculated by direct solution of the linear equation.

$$[I_h] = [Y_h][V_h] \quad (2.39)$$

where: $[Y_h]$ is the system admittance matrix at “h”. If the system is balanced only positive sequence admittances are used.

The admittance matrix must be formulated at each frequency of interest. The $[Y_h]$ must be re-formulated from scratch using the rules which apply to systems (with no mutual couplings between admittances), namely:

Rule 1: $Y_{ii} = \Sigma$ (all admittances connected to busbar i)

Rule 2: $Y_{ki} = - \Sigma$ (all admittances connected between busbars k and i).

The resultant voltages and currents are calculated by:

$$V_{RMS} = \sqrt{\sum_{h=1}^{\infty} V_{h(RMS)}^2} \quad (2.40)$$

$$I_{RMS} = \sqrt{\sum_{h=1}^{\infty} I_{h(RMS)}^2} \quad (2.41)$$

2.4.3 TOTAL HARMONIC DISTORTION

The measure commonly used for indicating the harmonic content of a waveform with a single number is called total harmonic distortion (THD) and can be calculated for either voltage or current [11]:

$$\text{THD}\% = \frac{\sqrt{\sum_{h=2}^{h_{\max}} M_h^2}}{M_1} \times 100\% \quad (2.42)$$

where: M_h is the rms value of the component h of the quantity M . (If $M=V$ or $M=I$, we express THD% as VTHD% or ITHD% respectively). M_1 is the fundamental quantity.

2.4.4 POWER AND POWER FACTOR

Power is the rate of change of energy with respect to time. The instantaneous power absorbed by a load is the product of the instantaneous voltage across the load and the instantaneous current into the load. There are three standard quantities associated with power [11], [30]:

- Apparent power S (voltamperes) is the product of the rms voltage and current.
- Active power W (watts) is the average of the instantaneous power.
- Reactive power Q (voltamperes-reactive) is the portion of the apparent power in quadrature with the active power.

For purely sinusoidal waveforms,

$$P = S \cos \theta \quad (2.43)$$

$$Q = S \sin \theta \quad (2.44)$$

$$S = \sqrt{P^2 + Q^2} \quad (2.45)$$

$$Q = \sqrt{S^2 - P^2} \quad (2.46)$$

Where: θ is the phase angle between voltage and current.

S is apparent power.

P is active power.

Q is reactive power.

Power factor (pf) is the ratio of active power to the power supplied (apparent power). The definition for power factor is:

$$\text{pf} = P/S \quad (2.47)$$

The formula for S (equation 2.45) is not true for non-sinusoidal waveforms, that is:

$$S \neq \sqrt{P^2 + Q^2} \quad (2.48)$$

This is because in non-sinusoidal conditions $S = V_{\text{RMS}} I_{\text{RMS}} \neq \sqrt{P^2 + Q^2}$.

This has prompted researchers to propose a new quantity called distortion volt-amperes (D) to provide the difference. The quantities P, Q, S and D are related as follows [11], [30]:

$$S = \sqrt{P^2 + Q^2 + D^2} \quad (2.49)$$

where:

$$S = V_{RMS} I_{RMS} \quad (2.50)$$

$$P = \sum_{h=1}^{\infty} P_{(h)} \quad (2.51)$$

where: $P_{(h)}$ is the active power for all harmonic frequencies.

$$Q = \sum_{h=1}^{\infty} Q_{(h)} \quad (2.52)$$

where: $Q_{(h)}$ is the reactive power for all harmonic frequencies.

thus:

$$D = \sqrt{S^2 - P^2 - Q^2} \quad (2.53)$$

Instead of introducing (D), the IEEE Working Group provides a new quantity called Non-Active Power (N). The quantity (N^2) is substituted for $Q^2 + D^2$ in equation (2.49), [14]:

$$S = \sqrt{P^2 + N^2} \quad \text{thus} \quad N = \sqrt{S^2 - P^2} \quad (2.54)$$

where:

$$N^2 = Q^2 + D^2 \quad (2.55)$$

The quantity (N) is introduced to minimize the changes from non-distorted to distorted waves. The similarity between equations (2.46) and (2.54) can be seen.

The term “displacement power factor (dpf)” is used to describe the power factor using the fundamental frequency component only.

$$\text{dpf} = P_1/S_1 \quad (2.56)$$

Using equations (2.50 and 2.51), “true power factor (pf)” for distorted systems is defined as:

$$\text{pf} = \frac{P}{S} \quad (2.57)$$

$$\text{pf} = \frac{\sum_{h=1}^{\infty} P_{(h)}}{(V_{\text{RMS}})(I_{\text{RMS}})} \quad (2.58)$$

2.4.5 SIZING CAPACITORS FOR PF CORRECTION

Industrial loads include inductive components and have a lagging dpf. A capacitor connected in parallel with an inductive load is called power factor correction. An appropriate size of capacitor cancels out the reactive power, increasing the dpf and decreasing the apparent power supplied to the combined load at the fundamental frequency. When sizing capacitors for power factor correction, the fundamental frequency component of the reactive power is used. Capacitors can only correct Q_1 [11].

2.4.6 HARMONIC SOURCES

Non-linear devices cause distortion in power systems and mostly manifest themselves as harmonic current sources. The direct harmonic solution described by equation (2.39), requires information about harmonic sources [13]. This information can be determined from either of, a, b or c below [11], [13]:

- Field measurements (waveforms and/or spectrums).
- Published data (spectrum) of device if measurement values not available.
- An ideal model that assumes that the harmonic content is inversely proportional to the harmonic number (h), namely:

$$|I_h| = \frac{|I_1|}{h} \quad (2.59)$$

The main individual contributor to power system harmonic distortion is the 3-phase bridge converter of which the 6-pulse type is the most common. The frequency domain representation of the ac current in phase “a” of a 6-pulse convertor (ideal model) is [12]:

$$\begin{aligned} i_a = \frac{2\sqrt{3}}{\pi} I_d & \left(\cos \omega_1 t - \frac{1}{5} \cos 5\omega_1 t + \frac{1}{7} \cos 7\omega_1 t - \frac{1}{11} \cos 11\omega_1 t \right. \\ & \left. + \frac{1}{13} \cos 13\omega_1 t - \frac{1}{17} \cos 17\omega_1 t + \frac{1}{19} \cos 19\omega_1 t - \dots \right) \end{aligned} \quad (2.60)$$

The following observations can be made from equation (2.60):

- The absence of triplen harmonics.
- The presence of harmonics (h) of orders $6k \pm 1$ for integer values of k.

- c. Those harmonics of orders $6k+1$ are of positive sequence.
- d. Those harmonics of orders $6k-1$ are of negative sequence.
- e. The rms value magnitude of the fundamental frequency is [12]:

$$I_1 = \frac{2\sqrt{3}}{\sqrt{2}\pi} I_d = \frac{\sqrt{6}}{\pi} I_d \quad (2.61)$$

where: I_d = dc current

- f. The rms magnitude of the h th harmonic is:

$$I_h = \frac{I_1}{h} \quad (2.62)$$

- g. Harmonics injected by a 6-pulse converter are:

$$h_{ch} = 6k \pm 1 \quad k = 1, 2, \dots, N \quad (2.63)$$

These harmonics (h) are called “characteristic harmonics (h_{ch})”.

Measurement results and/or published data on 6-pulse convertors usually show that I_h is not inversely proportional to the harmonic number (h). When modeling harmonic sources, measurement results should be used for the harmonics. In their absence, published data should be used. If measurement or published data is not available then the ideal model can be used. This is the approach followed to model harmonic sources in this thesis. How harmonic sources contribute to distortion in systems is discussed next.

2.4.7 COMPUTER STUDIES ON HARMONIC PENETRATION

A harmonic software program is usually used for conducting harmonic penetration studies. In simple terms, the calculation procedure is as follows [5]:

- a. The admittance matrix, $[Y_h]$ is built for each harmonic.
- b. The bus impedance matrix for each harmonic is calculated, $[Z_h] = [Y_h]^{-1}$. For the given harmonic source current $[I_h]$, the bus voltage $[V_h] = [Z_h][I_h]$ are obtained.
- c. Currents in the system are then calculated.
- d. The V_{RMS} , I_{RMS} , $VTHD\%$ and $ITHD\%$ values are determined.

2.4.8 HARMONIC IMPEDANCE SCAN STUDIES

Harmonic Impedance Scan studies are conducted to identify where resonance occurs in a system when one or more harmonic sources are present.

Studies conducted only at discrete frequencies (e.g., $h_{ch} = 6k \pm 1$, $k = 1, 2 \dots N$), as with harmonic penetration could completely miss an important resonance. A change in system configuration (e.g., addition of a new capacitor) could shift this resonance onto a harmonic number with disastrous consequences [12].

A scan provides an impedance plot at selected busbars over a range of frequencies (H_{pu}) [37]. This is achieved by injecting a $1\angle 0^\circ A$ current into a selected injection busbar. The voltage is then calculated at the busbars in the system. The impedance is then calculated by dividing the voltage at the busbar by the current at that point. At the injection bus, the impedance is called the driving point impedance (Z_D). The impedance at any other bus is called the transfer impedance. As the frequency is swept over the defined range (which includes the discrete harmonic frequencies injected by the harmonic source in the system) the resonance point(s) is seen. A sharp rise in impedance value indicates a parallel resonance whereas a dip in the frequency response curve implies a series resonance.

2.4.9 HARMONIC ANALYSIS SOFTWARE PACKAGE

In this thesis the Electrotek Harmonic Analysis Package [37] is used for all the simulation investigations. The package comprises two programs, "SUPERHARM" and "TOP". SUPERHARM contains a wide variety of device and source models and can solve both balanced and unbalanced three-phase systems. SUPERHARM uses TOP, The Output Processor to visualize the simulation results. Examples of the various types of models used to conduct a simulation study are given in Appendices 2, 3, 6, 7 and 8. More details on models can be found in the SUPERHARM User Guide. As motors have an effect on harmonic resonance peaks, their modeling is briefly discussed. In SUPERHARM the LINEARLOAD model is commonly used to model motors. In the software manual under the heading "LINEARLOAD" the equivalent circuit for a linear load is given and takes into account the effect of induction motors. The model provides a %Parallel and a %Series selection and states that when the Linear Load model is used, all motor load should be 100% series. This is applied in the case studies conducted in this thesis.

The LINEARLOAD model using the %100 Series selection accommodates motor loads without the necessity for using the INDUCTIONMOTOR model provided for by SUPERHARM. The INDUCTIONMOTOR model is used only when specific industrial motors are to be studied and all the motor parameters as required by the model are known. For planning purposes and where specifics are not available, the LINEARLOAD with 100%Series selection is used.

2.4.10 HARMONIC RESONANCE

If a scan shows a resonance at one or more of the discrete frequencies of the harmonic source applied to the system, then harmonic resonance occurs.

The condition for resonance for a single “L” and “C” at a harmonic is stated as follows:

$$h\omega L = 1/h\omega C \quad (2.64)$$

Systems which, have two or more inductances and capacitances, show a frequency response, which has multiple resonance points [15].

2.4.11 HARMONIC EFFECTS ON POWER SYSTEM DEVICES

Details of the effects of harmonics on power system devices are given in the IEEE 519 standard. The main effects are [16]:

Transformers

- a. Increased copper losses $\left(\sum_{h=2}^{\infty} I_h^2 R \right)$
- b. Increased iron losses.
- c. Insulation stress.

Rotating Machinery.

- a. Increased heating due to copper and iron losses.
- b. Pulsating torque is produced due to the interaction between the magnetic fields of the fundamental and harmonic components.

Capacitor Banks

- a. Increased dielectric losses [12].

2.5 SUMMARY

To improve power system efficiency, the installation of power factor correction capacitors is common practice. When sizing capacitors for power factor correction the fundamental frequency component Q_1 is used as capacitors can only correct Q_1 . With a daily increase in the number of harmonic sources being installed and the increasing awareness of harmonic effects and the possibility of system capacitor resonance occurring, it is essential to understand the principles of resonance and harmonic analysis. For this reason the main concepts relevant to the research topic have been reviewed in this chapter.

CHAPTER 3

DECISION ANALYSIS

The chapter reviews the basic framework for investigating decision problems. The general structure of a quantitative (decision) model is explained, models are classified and general guidelines for modeling are introduced. The traditional decision theory process is surveyed. The process is represented as a block diagram and is shown to be made up of two stages, a decision model stage and a decision making stage. The decision table as a tool to represent a quantitative model is introduced to structure an otherwise unstructured decision problem. The differences between certainty, uncertainty and risk situations are discussed and the need for the decision making stage is shown.

This chapter focuses on decision making under risk. The meaning of risk is introduced and as it depends on probability, the relevant principles and distributions of probability are reviewed. A distinction is made between discrete and continuous distributions as well as between objective and subjective assessment methods for eliciting probabilities. A general format for a decision table, together with the expected monetary rule (EMV) is discussed for making decisions under risk and where the outcomes are monetary values. The limitations of the EMV rule are discussed and the need for utility theory as an evaluation model is discussed. Utility theory is reviewed and a utility table and the expected utility value rule are introduced to make preferred decisions. The variable probability method and its basis in axiomatic theory of utilities are introduced to make decisions when outcomes have monetary and non-monetary values. Six worked examples are introduced to enhance understanding and to prepare the reader for the application of decision theory to the making of a mitigation decision on harmonic resonance.

3.1 BACKGROUND

People have to make decisions to solve problems for themselves and/or the organizations for which they are employed. Decision makers have to make a choice from numerous possible alternative courses of action. Generally, they find this difficult because they cannot handle all the factors influencing the decision and they also do not know how to evaluate the results obtained from the numerous possible alternatives.

3.2 GENERALIZED FRAMEWORK

A generalized framework has been developed to serve as a foundation for investigating all types of decision problems and has three main steps [24]:

1. Define the objective.
2. Determine the controllable and uncontrollable inputs.
3. Find the best choice by determining the effect that the controllable input has with respect to the objective, taking constraints into account.

The first step is to define an objective (result variable) for the decision problem to be solved (e.g., profit). Once the objective has been defined, the problem becomes more complicated as factors influencing the decision are usually variables. These variables are called controllable and uncontrollable variables.

The action of making a decision consists of making a choice from numerous possible alternatives under the control of the decision maker (e.g., price to be charged for a product). These actions (variables) are called “decision alternatives” or “controllable inputs” as they are subject to the human decision process of choice.

Decision problems often contain variables that are not under the control of the decision maker and are called “uncontrollable inputs” or “state of nature variables” (e.g., demand for a product).

Most of the time the decision maker will be unable to select any value for his inputs since not all values are feasible. He will therefore “constrain” his selection of inputs to feasible values only.

The decision maker will determine the effect of each possible decision alternative on the objective, taking into account the uncontrollable inputs and constraints. This means, finding the best choice by observing the results.

3.3 NEED FOR A MODEL

The process is more complicated than the outline suggests. The steps hint at a procedure, which involves relationships between the three variables.

Traditionally, decision makers resort to rough guesses, estimates and simplifying assumptions and this can lead to inaccurate results. Therefore, without the aid of quantitative techniques (mathematical relationships) the outlined process is inadequate for complicated decision problems.

3.4 GENERAL STRUCTURE OF MODEL

Making good decisions is not an easy task most of the time. It has been found that the foundation for making a sound decision is enhanced by building a mathematical model for the decision problem to be analyzed [17].

By building a quantitative (decision) model, we learn the basics of decisions, how models are constructed, how they are used and what they can tell us. Modeling allows the decision maker to address the most important issues, that is, determining what fundamental questions to ask, what alternatives to investigate and where to focus attention. A model will therefore offer direct support and will lead to better understanding of the decision problem [18].

Mathematical models are abstract as all concepts are represented by quantitatively defined variables and where all the relationships are mathematical instead of physical or analog. The defined variables are interrelated by equations and facilitate experimentation and analysis [18]. All quantitative models, including decision models have the following structure [18], [23], [24]:

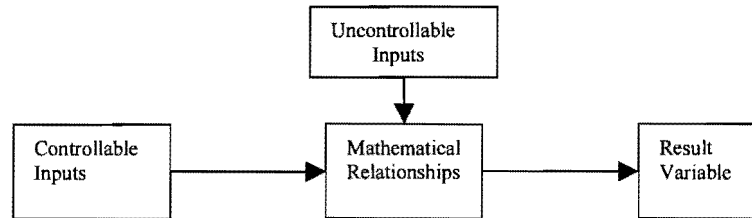


Figure 3.1 General structure of a decision model

The first and most important challenge when faced with a decision problem is to define the specific objective (result variable, outcome). Next, the controllable and uncontrollable inputs are determined and represented symbolically. The most challenging aspect involves developing equations to describe the decision problem. The most important equation is the one that relates the objective (payoff measure) to the controllable and uncontrollable inputs and is called the objective function for the decision problem. It is used as the basis for evaluating the choices (decision alternatives) introduced into the model and takes constraints into account.

3.5 CERTAINTY, UNCERTAINTY AND RISK CLASSIFICATIONS OF DECISION PROBLEMS

When some of the factors of a decision problem are known, we say that for these aspects a situation called “certainty” exists (e.g., cost of a product). A situation called a state of “uncertainty” exists when the decision maker knows nothing about the likelihood (probability distribution) of the uncontrollable inputs (e.g., demand). If the decision maker is able to objectively or subjectively assign a value to the demand, for instance, then the situation is called “risk”.

The analysis of the decision problem is therefore dependant upon the situation of the variables influencing the model.

3.6 GENERAL GUIDELINES TO MODELING

There are three general guidelines for decision modeling [19]:

a) GUIDELINE 1

Nothing enhances ability to solve a problem more than developing an eye for data relevant to the objective sought. The definition of variables is an important step in formulating a model.

Developing the equation (objective function) to describe the payoff measure (outcome) and its mathematical relationships to the other variables in the model is the most important step in the modeling process.

An early step in modeling is thus to determine what data is relevant to the objective sought. It is essential to choose variables that lead to a solution of the decision problem.

b) GUIDELINE 2

A good model is an informed approximation but cannot be an exact replica of reality. In model building certain assumptions are made. However, if skillfully built, the simplifications have value as they clear away details that are unimportant and focus on the key effects.

If the values of some of the variables are not meaningful, we must include in the model, equations that restrict the variables, so that they take on only meaningful (feasible) values. These restrictions are called “constraints”.

Thus, when formulating a model, the builder must isolate/select those aspects of the data relevant to the problem at hand. Such a model is called a constrained quantitative model and has an objective function subject to one or more constraints [19].

In essence one should use a model that will give a meaningful (feasible) answer to the decision problem.

c) GUIDELINE 3

The purpose of modeling is to obtain insight and understanding, not solely numbers. Therefore a model must be aimed at quantitative and qualitative insights that help bring an issue into focus.

The numerical result variable (outcome) of the model must be able to be expressed qualitatively (e.g. profit). A model must therefore give a decision maker insight and understanding about the decision problem faced.

3.7 CLASSIFICATION OF A MODEL

Mathematical models are classified in various ways.

- a. Optimization models maximize or minimize a quantity [21].
- b. When the function of a model is not to maximize or minimize a quantity, but to predict outcomes they are called prediction models [21].

- c. Depending on the information available, the values of the uncontrollable inputs may be known or uncertain. If they are known the model is said to be a “deterministic model”, if unknown (uncertain) then it is a “stochastic model”.

3.8 DECISION THEORY PROCESS

Although the insight and understanding gained by modeling decision problems can be helpful, decision making often remains a difficult task, especially when an uncertain or risk situation exists. Therefore, besides the quantitative model, additional scientific tools are needed to assist with decision-making [17].

To overcome this difficulty, management scientists have developed a rational methodology for conceptualising, analysing and solving all types of decision-making problems [23].

This approach is referred to as “**decision analysis**” or the “**decision theory process**”[23].

The process has two stages:

- a. Stage 1: A quantitative model building stage.
- b. Stage 2: A decision making stage.

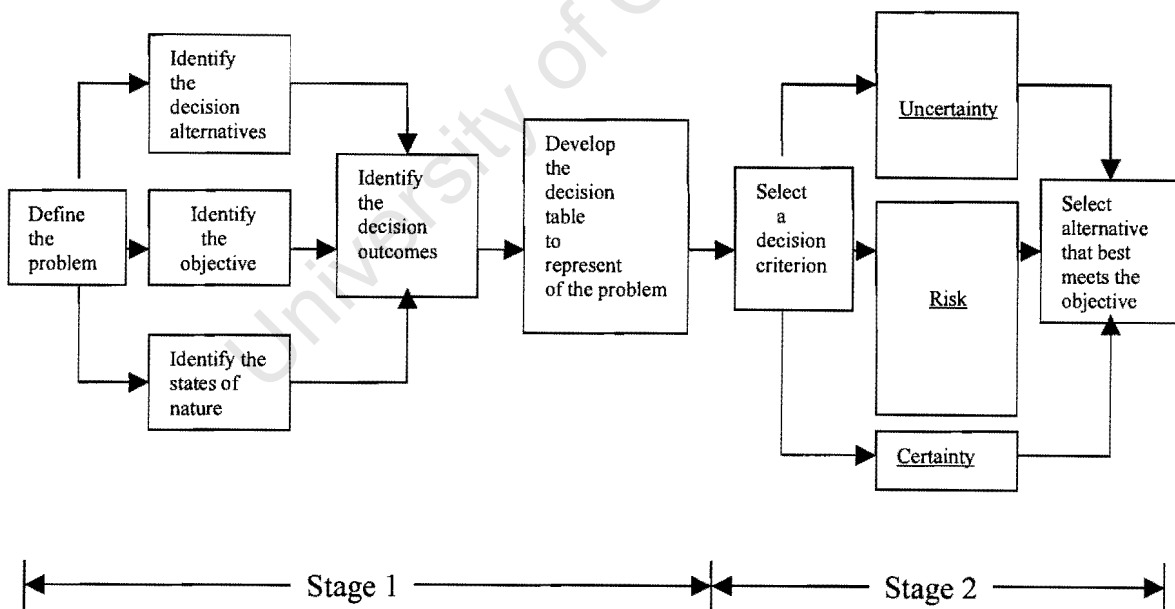


Figure 3.2 Decision Theory Process

3.9 MODEL BUILDING STAGE

The steps involved in building a quantitative (decision) model (except for the decision table), were explained in sections 3.1 to 3.8. For more clarity, the following model building characteristics are added.

- a. In any moderately complex decision the potential number of choices can be unlimited. The nature of the problem will usually limit (constrain) the number of available actions, so that the controllable inputs (decision alternatives) are meaningful (feasible). This means that the number of alternatives could be constrained to a narrow range [17], [23].
- b. When modeling, it is essential that a relevant set of decision alternatives are identified. Therefore, exercising good judgement is especially important in this preliminary stage. Only those alternatives, which the decision maker wants, need be included [25].
- c. The main goal of decision analysis is to allow the decision maker to select a decision from a set of alternatives when uncertainties regarding the future exist [21]. These uncertainties, that is, the conditions that are expected to occur, need to be identified. In decision theory, these events (uncontrollable inputs) are called states of nature.

In uncertain situations, there are an unlimited number of possible events, especially when they are numerically expressed. However, the problem format will usually permit the number of states of nature to be constrained to a very narrow finite range of values. For example, the demand for a toy may be uncertain, but the manufacturer feels that one of the following events is likely to occur [25]:

Light demand	(25 000 units)
Average demand	(100 000 units)
Heavy demand	(150 000 units)

In this example, the toy manufacturer is only considering three possible events for demand instead of all possible events. In decision theory, we often use a small discrete set of events to represent states of nature [17]. The decision maker has also described the possible events to be considered in the analysis in qualitative categories (light, average, heavy) besides quantitative categories (25 000, 100 000, 150 000).

- d. The relevant set of decision alternatives and the set of events, each need to be defined in a way that precludes any two (or more) alternatives or events from occurring simultaneously. The occurrence of one will then exclude the other. In decision theory, the decision alternatives and the states of nature must each be mutually exclusive [23], [25], [26].

For example: If events A and B have no points in common, then,

$$A \cap B = \phi$$

where: ϕ is the empty set (set with no elements, there is no intersection of A and B).

Then, we call A and B mutually exclusive (or disjoint), as only one of them can occur in any single trial of an experiment.

- e. Further, the relevant set of events for a decision experiment must also be collectively exhaustive, making it certain that one of the events will occur.
- f. To evaluate the choices, the decision maker must measure the outcome that will result from each possible combination of decision alternatives and states of nature. These outcomes, called payoff's or decision outcomes are equivalent to the result variable of the decision model and must mathematically relate the alternatives and states of nature to each other and be relevant to the objective.
- g. This model building approach has been developed to deal with a decision problem which has the following specific characteristics [24]:
 - (1) A decision maker has to make his decision from among a “set” of defined decision alternatives (a_1, a_2, \dots, a_n).
 - (2) A “set” of events (s_1, s_2, \dots, s_N) exists that are not under the control of the decision maker (states of nature).
 - (3) For each alternative (a_1, a_2, \dots, a_n), a determinable outcome (result variable, $r_{11}, r_{12}, \dots, r_{nm}$) will result, that is, conditional on an event (s_1, s_2, \dots, s_N) occurring.

This approach is not limited to only uncertain situations but also is used as a basis when situations of certainty and risk are to be considered.

3.9.1 DECISION TABLE

When a decision maker has to make a decision from a finite set of discrete decision alternatives, whose outcome is a function of a single future event, a “**decision (payoff) table**” analysis is the simplest manner of formulating the decision problem [18].

A decision table (payoff matrix) is a table that summarizes the final outcome (payoff) for each alternative under each possible state of nature [17].

In this way, a decision table provides a framework for structuring one-time decision problems and indicates the relationship between pairs of decision elements [23], [25].

In decision theory, quantitative models are represented by means of a decision table. The general form of a decision table is [18]:

DECISION ALTERNATIVES	STATES OF NATURE			
	s_1	s_2	...	s_N
a_1	r_{11}	r_{12}	...	r_{1m}
a_2	r_{21}	r_{22}	...	r_{2m}
\vdots	\vdots	\vdots	\vdots	\vdots
a_n	r_{n1}	r_{n2}	...	r_{nm}

Table 3.1 Decision Table

For a given problem, the decision table lists the states of nature (mutually exclusive and collectively exhaustive) across the top and the decision alternatives (mutually exclusive) down the left-hand side of the table. The values of the outcomes are given in the body of the matrix and must be in consistent units and are the payoffs for all combinations of decision alternatives and states of nature. An example of how to structure and represent a decision problem as a decision table is given in reference [24].

A decision table has basically two disadvantages:

1. It does not tell the decision maker how to choose the decision alternative that best meets his objective.
2. The representation of the model as a payoff table does not make decisions, people do.

The structure of the decision table therefore does not suggest how decisions should be made. How to make a decision is the subject of the next stage in the decision theory process. In this second stage, called the decision making stage we will consider the following three situations.

SITUATION 1: decision making under certainty.

SITUATION 2: decision making under uncertainty.

SITUATION 3: decision making under risk.

3.10 DECISION MAKING STAGE

The question posed in the last section was, how does the decision maker choose the decision alternative that best meets his needs?

This choice is subject to knowledge which the decision maker has about the states of nature. This knowledge is divided into three zones [23]:

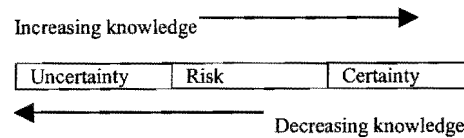


Figure 3.3 Decision situations

Figure 3.3 illustrates the range of decision situations. As can be seen, one can move from uncertainty towards a certainty situation by increasing one's knowledge. By increasing one's knowledge about the environment, a more informed decision can be made.

At one extreme of the range of states of nature, the decision maker can identify possible future conditions (e.g. demand). However, he does not have sufficient information to assess the likelihood of each of the states of nature relevant to the decision problem. This situation is called uncertainty and is undesirable but occasionally unavoidable. Uncertain situations occur when there is a completely new phenomenon (e.g., space travel, a novel product, new technology).

At most times, there is some information available that can be used to at least quantify the uncertainty.

A source of this information is the decision maker himself. Also managers, project and design engineers possess intuition and during their careers accumulate knowledge based on work experience. Such decision makers can use their experience to subjectively assign probabilities to each state of nature. Such judgments predicting the likelihood of states of nature are called subjective probabilities. In many situations, future conditions can be expected to follow the same pattern as past events. Such can be used to calculate the proportion of times that each state of nature was observed in the past and is called objective probability.

When the likelihood of states of nature can be established, knowledge has increased and the decision moves from one of uncertainty to the risk zone. The situation is then referred to as decision making under risk.

At the other extreme on the range, if enough knowledge (information) has been acquired to know exactly which state of nature will occur, then the situation is one of certainty.

Having to make decisions (select a choice from the set of decision alternatives) under the uncertainty situation is not an everyday occurrence, as it is not often that something totally novel presents itself. At the other extreme of the range, the certainty situation is also not as prevalent but does occur. Of the three situations, we are most likely to encounter decision making under risk [18]. For completeness, the two extreme situations will be reviewed briefly and this is followed by decision making under risk.

3.11 DECISION MAKING UNDER CERTAINTY SITUATIONS

A decision under certainty occurs when you know which state of nature will happen. Alternatively, it can be seen as a case (decision table) with only a single state of nature (one column). An example when a certainty situation exists is given in Appendix 9.

3.12 DECISION MAKING UNDER UNCERTAINTY SITUATIONS

Here we have more than one state of nature and the decision maker is unable to assign probabilities to them.

There has been a debate for many years as to whether such a situation should be allowed to exist. The argument is that the decision maker should always be willing to at least subjectively assign probabilities to the states of nature [18].

If no probabilities can be assigned to the states of nature we are dealing with strict uncertainty. The method of solution is to use one of the following decision criteria [23]:

- a. Optimistic criterion.
- b. Pessimistic criterion.
- c. Coefficient of optimism criterion.
- d. Regret criterion.
- e. Rationality criterion.

Table 3.2 gives a comparison of the criteria used for decision making under uncertainty:

Optimism	Pessimism	Coefficient of Optimism	Regret	Rationality
Identify the most favourable outcome associated with each decision alternative.	Identify the worst outcome associated with each decision alternative.	Identify both the best and worst outcomes associated with each decision alternative.	Calculate the regret associated with each state of nature. Regret is the difference between the best possible payoff and the outcome actually received from selecting a decision alternative.	Compute the simple average outcome for each decision alternative. The simple average is the sum of outcomes divided by the number of events.
Select the alternative that leads to the best of the most favourable outcomes.	Select the alternative that leads to the best of the worst outcomes.	Identify the decision-maker's coefficient of optimism.	Identify the largest regret associated with each decision alternative.	Select the decision alternative that leads to the best of these simple average outcomes.
		Calculate the weighted outcome for each decision alternative.		
		Select the alternative that leads to the best weighted outcome.	Select the alternative that leads to the smallest of largest regrets.	

Table 3.2 Comparison of criteria for decision making under uncertainty

It has been found that these criteria lead to different decisions for the same problem [23], [24].

The ambiguous nature of decision-making under uncertainty, limits our ability to analyze decision problems. For this reason, it is not discussed further but if more on this topic needs to be known the reader is referred to literature in the field [17] - [25].

Increasing our knowledge about the environment will enable us to make more informed decisions.

That is why most decision makers do not make a final decision until enough information has been acquired to at least measure the uncertainty with probabilities [22].

Decision-making under risk is therefore the most relevant topic and is discussed next [18].

3.13 DECISION MAKING UNDER RISK SITUATIONS

3.13.1 MEANING OF RISK

The term “risk” has a restrictive and well-defined meaning, namely:

Decision-making under risk refers to a decision model for which there is more than one state of nature and the decision maker can arrive at a probability estimate for the occurrence of each of the various states of nature [18].

3.13.2 APPROACHES TO PROBABILITY

Probability principles and distributions are briefly reviewed, as decision makers need to assign probabilities to the occurring states of nature (e.g., demand) in their developed decision table.

Probability is termed the mathematical language of uncertainty. It represents a means to measure and quantify uncertainty.

There are three approaches to deriving probabilities [20]:

1. Classical Approach.
2. Relative Frequency Approach.
3. Subjective Approach.

The first two approaches are normally referred to as objective probabilities. If different people have access to the same information, they should arrive at exactly the same probabilities.

In the subjective approach different probabilities could be put forward.

3.13.3 CLASSICAL APPROACH

Classical (a priori, before the fact) probabilities are related to games of chance. When elementary events are equally likely, then the probability (p) that a given event (E) occurs is the ratio of the number of elementary events included in E to the number of elementary events in the sample space S , namely [27]:

$$p(E) = \frac{\text{Number of elementary events in } E}{\text{Number of elementary events in } S} \quad (3.1)$$

In order to apply this approach to a problem, it has to be assumed that each outcome is equally likely to occur. In most practical situations the outcomes are not equally likely to occur, therefore the usefulness of this approach is limited.

3.13.4 RELATIVE FREQUENCY APPROACH

The relative frequency approach (a posteriori, after the fact) is based on the concept that the probability of an observed event is equal to the relative frequency of the actual occurrence of that event in the long run. Judgements are based on after the fact information and based on empirical data. For example, a quality control inspector at a factory might test 250 light bulbs and find that 10 are defective. This suggests that the probability of a bulb being faulty is $10/250 = 0.04$ (4%). For this estimate to be reliable the same manufacturing process would have to apply to every week under consideration. If there were any change then the estimate would not be reliable. The relative frequency approach to probability is very useful and is used in most cases where empirical data is available. The data must however relate closely to the event under consideration.

3.13.5 SUBJECTIVE PROBABILITY

In some situations the objective probability approach based on relative frequency cannot be used due to a unique event occurring, that is, one that has never occurred before.

For example, an electrical capacitor is planned to be installed into a power system. Due to uniqueness of the situation, no past data on the resonance effects is available, thus the relative frequency approach to estimating probabilities does not apply. Capacitors have in the past been installed in power systems, but it is unlikely that the conditions that applied then, will be directly relevant to the current problem. In such circumstances the probability is estimated using the subjective approach [20].

A subjective probability can be interpreted as a measure of the degree of belief that a particular event will occur. It is an expression of an individual's degree of belief that a particular event will occur. Because of the unavailability of suitable statistical data, probabilities are subjective estimates based on human judgement. This raises the question, how good are people at estimating probabilities?

This topic has been researched for the past thirty years and it has been found that when assessing subjective probabilities [20]:

- a. An attempt must be made to locate a reference class of previous forecasts that was made which are similar to the event that needs to be forecasted and feedback on its accuracy determined.
- b. If not previously done by the decision maker, then he should consider whether there is a historic, relative frequency reference class that could be used.
- c. If a reference class of historic frequencies is not obvious, then judgemental heuristics need to be used taking into account bias.

These findings are summarized as follows [20]:

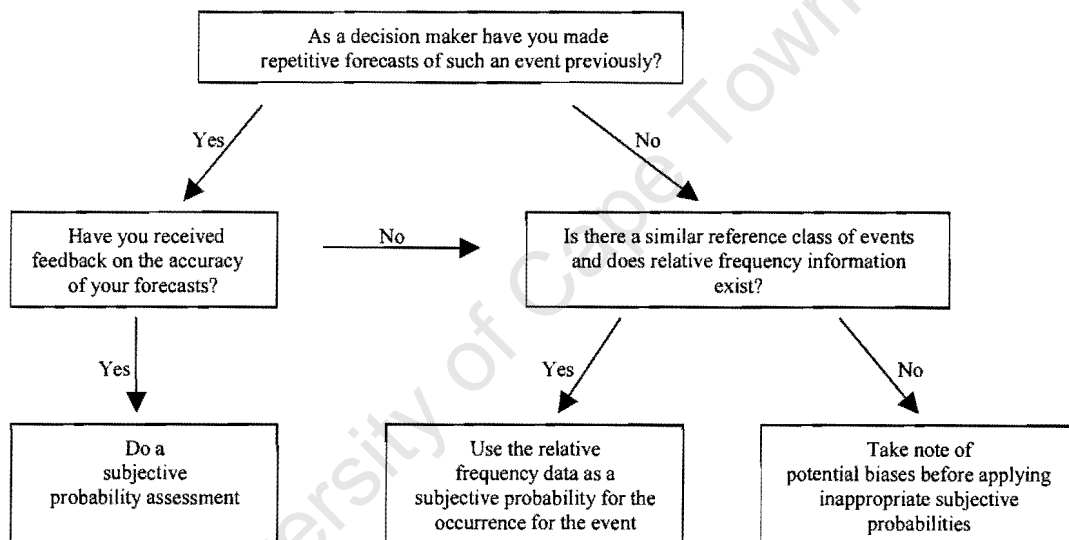


Figure 3.4 Subjective probability assessment

3.14 PROBABILITY DISTRIBUTIONS

A statement of all possible events and their probabilities is known as a probability distribution [20]. There are two types of distributions:

1. Discrete Probability Distributions.
2. Continuous Probability Distributions.

If only a finite number of states of nature (events) are possible then we refer to discrete probability distributions. In contrast, in a continuous probability distribution the uncertain quantity can take on any value within a specified range.

3.15 DISCRETE PROBABILITY DISTRIBUTIONS

Any discrete probability distribution can be represented by a probability function. For any given random variable (x), the probability function is an enumeration of each possible value that can occur and its associated probability.

Assuming x_1, x_2, \dots, x_n values for x and p_1, p_2, \dots, p_n values for associated probabilities (p) the probability function can be written as:

$$p(x_i) = p_i \quad i = 1, 2, \dots, n \quad (3.2)$$

where: $p(x_i)$ is the probability that x assumes a specific value x_i .

It must satisfy two conditions:

$$p(x_i) \geq 0 \quad (3.3)$$

and, individual probability values must sum to 1.0, so:

$$\sum_{i=1}^n p(x_i) = 1 \quad (3.4)$$

The general properties of any probability distribution of a random variable (x) associated with a finite sample space are:

$$1. \quad 0 \leq p(x) \leq 1 \quad x \in \{x_1, x_2, \dots, x_n\} \quad (3.5)$$

$$2. \quad p(x_1) + p(x_2) + \dots + p(x_n) = 1 \quad (3.6)$$

If (x) is an event, then “(x) does not occur” and is said to be the complement of the event (x) and is written as \bar{x} .

This leads to the following general expression [20]:

$$p(\bar{x}) = 1 - p(x) \quad (3.7)$$

Probability values must thus fall within the range of 0 to 1, thus [22]:

- Probability = 0 means the event will never occur.
- Probability = 1 means the event will always occur (certainty).
- Probability, $0 < p(x) < 1$ gives relative frequency.
- Probability outside 0 and 1 range have no meaning.

3.15.1 OBJECTIVE ASSESSMENT METHOD

From the objective approach for probabilities, the probability of an event is a measure of its relative frequency or likelihood of occurrence.

A method for describing this data graphically is a frequency distribution.

With the aid of an example, the objective assessment method for eliciting probabilities from a discrete probability distribution is demonstrated.

3.15.1.1 EXAMPLE 3.1 – HISTOGRAM FOR ELECTRIC LAMP ORDERS

Historic data on mail orders for a certain electric lamp is shown in table 3.3 [27]:

FREQUENCY DISTRIBUTION		
Orders (d)	Frequency (number of days)	Relative frequency
0	14	0.14
1	25	0.25
2	48	0.48
3	13	0.13
-	=100	-

Table 3.3 Frequency Distribution for Electrical Lamps

Here, the random variable (daily lamp orders), “d” (demand) represents a specific number of orders.

Table 3.3 shows the daily lamp orders over a period of 100 days. For example, in a 25day period there will be one order received for electric lamps.

If we assume that demand does not change, we can use “past experience” as a guide and use the relative frequency as probabilities for the number of lamps ordered, thus,

$$\begin{array}{r}
 p(0) = 0.14 \\
 p(1) = 0.25 \\
 p(2) = 0.48 \\
 p(3) = 0.13 \\
 \hline
 1.0
 \end{array}$$

The discrete probability distribution (histogram) based on objective probabilities (relative frequency) is thus as follows:

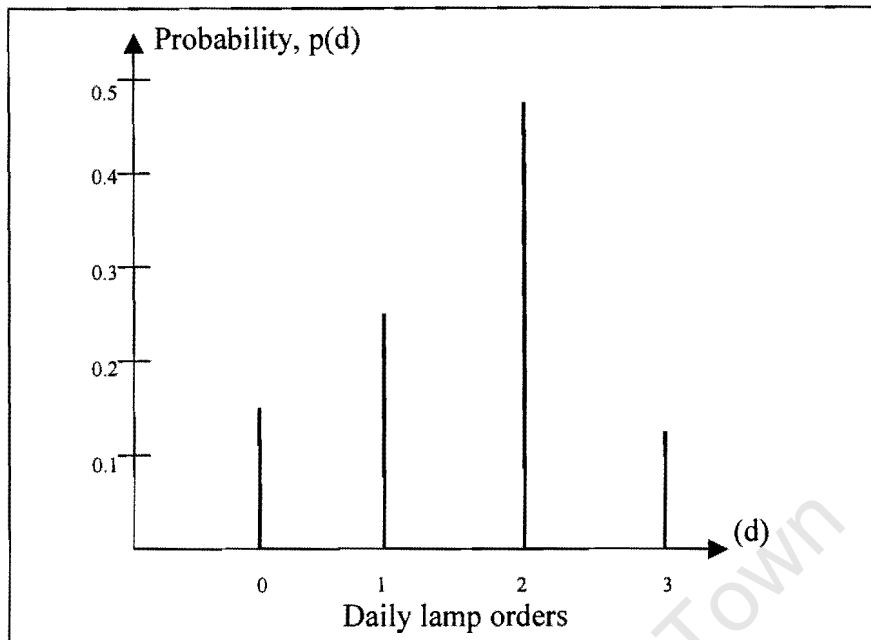


Figure 3.5 Probability function (Histogram).

Each probability value appears as a vertical line.

3.15.2 SUBJECTIVE ASSESSMENT METHOD

Suppose we are attempting to subjectively estimate the demand for a new 2000kVA plant not yet in operation. The objective probability approach based on relative frequency cannot be used due to this being a unique event that has not occurred before.

If the number of possible values of the random variable representing demand is relatively small (e.g., 10 or less), a subjective discrete probability distribution can be structured based on a degree of belief that each possible value of random variable (demand) will occur.

The “method of relative heights” is a graphical technique that is designed to elicit a probability density function (pdf) [20]. Using the method of relative heights, the decision maker is asked to identify the most likely value of the random variable (demand) under consideration. A vertical line is drawn on a graph to represent this likelihood. Shorter lines are then drawn for the other possible values to show how their likelihood’s compare with that of the most likely value.

3.15.2.1 EXAMPLE 3.2 – DISCRETE PROBABILITY DISTRIBUTION FOR AN ELECTRICAL PLANT

Let (s_d) represent the random variable denoting the electrical demand for the new 2000kVA plant.

After questioning the engineer for the new 2000kVA plant, he reveals, based on past experience of the operation of other plants, that there are only three possible events to be considered.

1. 2000kVA (100% full load demand).
2. 1200kVA (60% full load demand = average demand).
3. 500kVA (25% full load demand).

After further questioning, the engineer reveals that the 1200kVA demand is the most likely demand level. This then is represented on a graph by a line of 10 units long.

He further reveals that 500kVA is half as likely as 1200kVA. This is represented as a line of 5 units on the graph. Still further questioning, reveals that 2000kVA is one-tenth as likely as 1200kVA, and is represented on the graph by 1 unit.

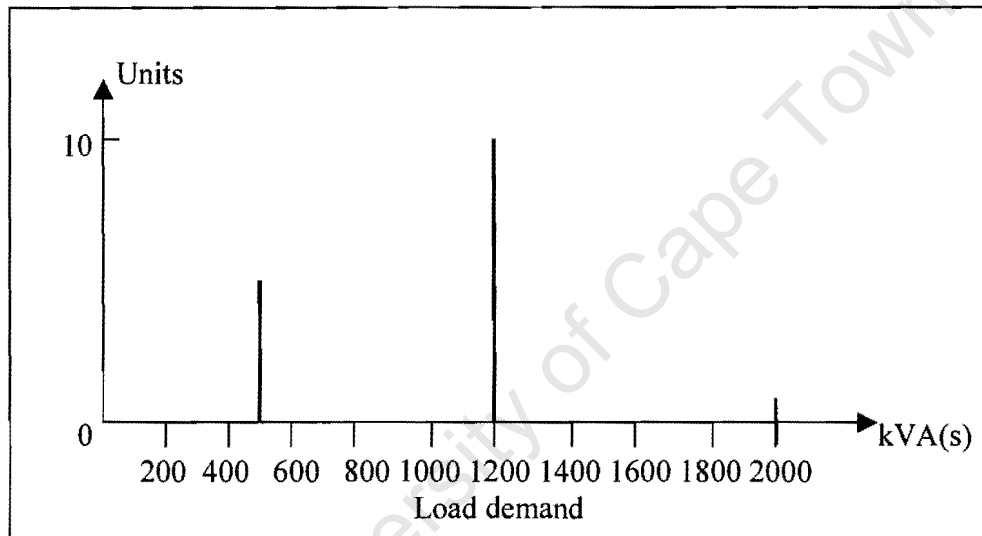


Figure 3.6 Frequency distribution

Since the vertical lines represent probabilities, their probability values must sum to 1.0. Establish the height of each line and then divide its value by the sum of all these heights to get its individual probability value.

This process of summing individual values and the division by each such value by that of the sum is called the “normalization process”. The sum of such normalized values must equal 1.0. In this example, the sum of the line lengths is $5 + 10 + 1 = 16$. Therefore the probabilities are:

$$\begin{aligned}
 p(500) &= 5/16 = 0.3125 \\
 p(1200) &= 10/16 = 0.6250 \\
 p(2000) &= 1/16 = 0.0625
 \end{aligned}$$

$$1.000$$

If we round off to one place after the decimal point,

$$\begin{array}{rcl} p(500) & = & 0.3 \\ p(1200) & = & 0.6 \\ p(2000) & = & 0.1 \\ \hline & & 1.0 \end{array}$$

Therefore, the discrete probability distribution is:

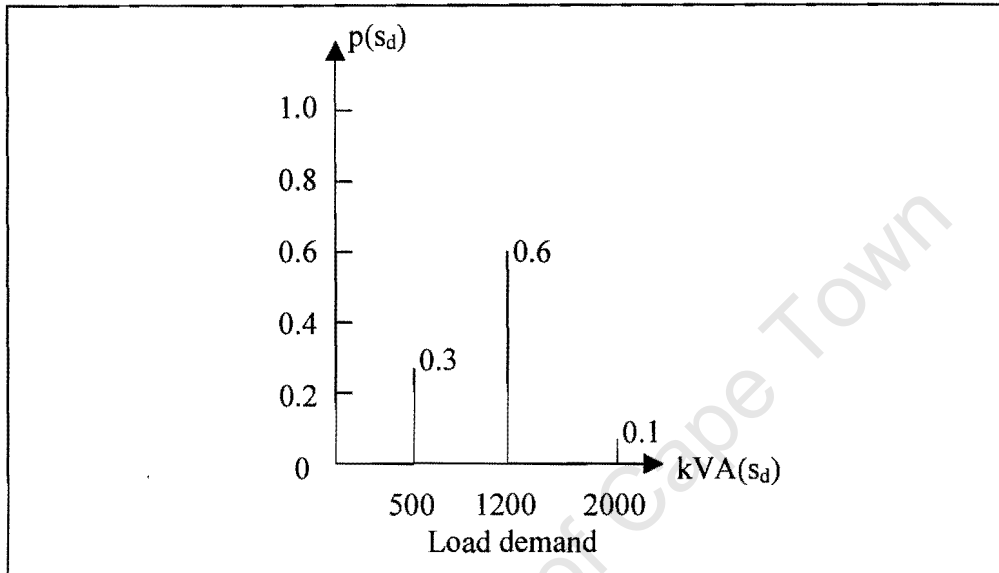


Figure 3.7 Discrete probability distribution

3.16 CONTINUOUS PROBABILITY DISTRIBUTIONS

In a continuous probability distribution the uncertain quantity can have any value within a specified range [20], [27].

Temperature at a certain electrical plant is a typical example of a continuous random variable as the concept of associating a probability with each possible value of the random variable is no longer meaningful. Instead, we rather refer to the probabilities that the random variable falls within a given temperature range.

A continuous probability distribution is merely a discrete probability distribution with a very large number of values close to each other. Like with discrete probability distribution, continuous probability distributions can also be assessed objectively or subjectively.

3.16.1 EXAMPLE 3.3 - OBJECTIVE ASSESSMENT METHOD

The following data is known about the temperature (random variable x), at a certain electrical plant, from which objective probability values are calculated [27].

FREQUENCY DISTRIBUTION FOR TEMPERATURE		
Temperature (°C)	Number of observations (frequency)	Probability
0 to under 2	553	0.0553
2 to under 4	1066	0.1066
4 to under 6	2090	0.2090
6 to under 8	3033	0.3033
8 to under 10	1885	0.1885
10 to under 12	1025	0.1025
12 to under 14	348	0.0348
	<u>10000</u>	<u>1.0000</u>

Table 3.4 Frequency Distribution for temperature at an electrical plant

The probability data is then plotted in a histogram.

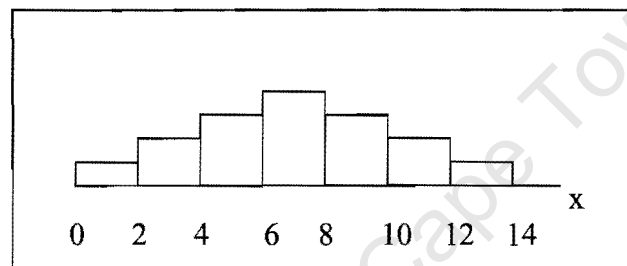


Figure 3.8 Histogram for temperature at an electrical plant

The histogram has an even class interval (0 to under 2, etc.) and the height of each rectangle is proportional to the frequency of x . If the vertical axis is scaled in such a manner so that the area of all of the rectangles sum to 1.0, then the area of each individual rectangle will be equal to the probability that x falls within the given class interval. Therefore, the area of the rectangle covering the class interval from 6 to 8 represents the probability that (x) falls between 6 and 8.

Suppose, the class intervals are made very small but the total area is constrained to 1.0, so that each rectangle still represents the probability that (x) falls in the class interval, as follows:

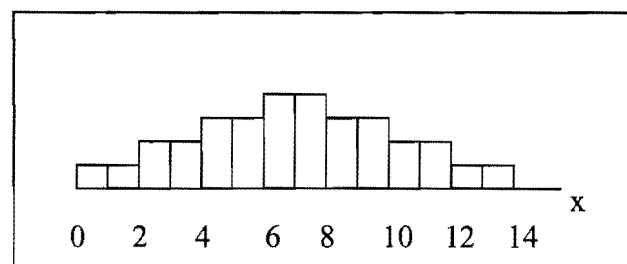


Figure 3.9 Histogram with narrow class intervals

If the intervals are made small so that they vanish, but keeping the total area under curve = 1.0, we arrive at a continuous curve called the probability density function (pdf), $f(x)$, for a continuous random variable (x).

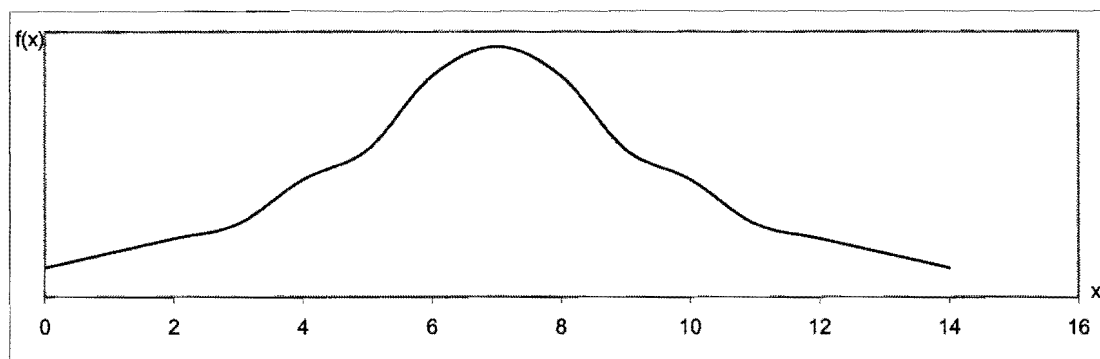


Figure 3.10 Probability density function

A pdf must satisfy two requirements:

1. $f(x)$ is always non-negative.
2. the area under the curve $f(x)$ must equal 1.0.

For any value of (x), the value $f(x)$ does not represent the probability that the value (x) occurs as was the case with discrete probability distributions. A so-called point probability is meaningless with continuous probability distributions as there is no area under a point. Thus, with continuous probability distributions we refer only to the probability that (x) falls within a given range. Taking this a step further, the cumulative distribution function (cdf) of (x), denoted by $F(x)$, is defined as the probability that (x) is less than or equal to some specific value (x), then,

$$F(x) = p(X \leq x) \quad (3.8)$$

Now, by cumulating the probability density function values $f(x)$ of figure 3.10, the cdf is:

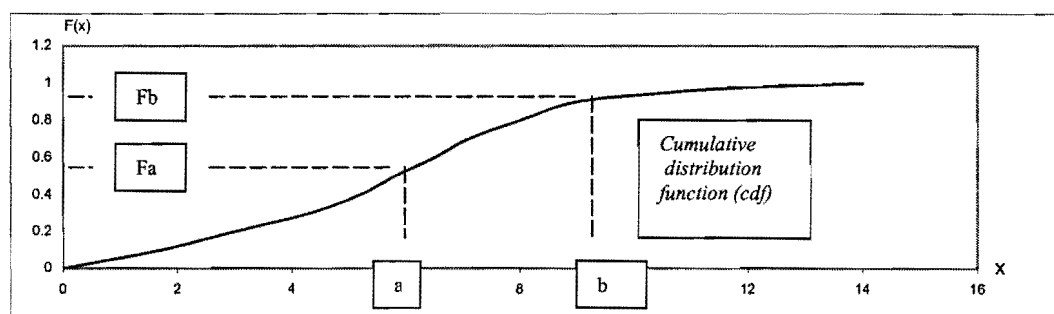


Figure 3.11 Cumulative distribution function

The most useful property of a cdf is that it enables one to readily evaluate the probability that the random variable (x) lies within a class interval (a, b), $b > a$, that is:

$$p(a \leq x \leq b) = F(b) - F(a) \quad b > a \quad (3.9)$$

3.16.2 EXAMPLE 3.4 - SUBJECTIVE ASSESSMENT METHOD

The “method of relative heights” is also effective for assessing probability density functions for continuous distributions. The relative likelihood of a few values is evaluated and then a smooth curve is fitted across the tops of the vertical lines on the graph [20].

Considering the same situation as in example 3.2, that is, the engineer is required to estimate the weekly demand for the new 2000kVA electrical plant.

The engineer believes there is a very small chance (say, 1%) that demand will be less than 250kVA or more than 2000kVA.

Let, (s_d) be the random variable denoting the electrical demand.

The first step in assessing subjective probabilities is to construct a bar chart (histogram) by asking the engineer to estimate the probability that demand falls within a given range.

Next, divide the range into 7 intervals, so that there is 250kVA per interval: 250 – 500, 500 – 750, 750 – 1000, 1000 – 1250, 1250 – 1500, 1500 – 1750, 1750 – 2000, respectively.

Using the method of relative heights, after questioning, the 1000 – 1250 interval is the most likely demand level. This is represented on a histogram by a bar of height 10 units.

Further questioning of the engineer reveals that the 500 – 750 interval is the next likely demand and is half as likely (50%) as the 1000 – 1250 demand level. This is represented on the graph as 5 units.

Still further questioning reveals that the 1750 – 2000 interval will be one-tenth (10%) as likely to occur as the 1000 – 1250 demand level and is represented on the bar graph as 1 unit.

The demand levels for the other four intervals are likely to have only 2% values. They are represented as bars by 0.2 units, respectively.

The histogram is as follows:

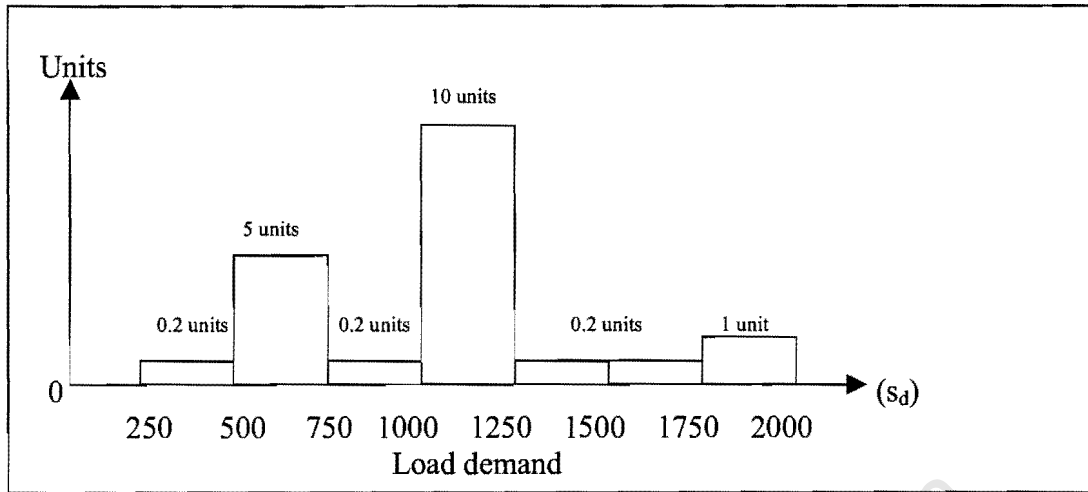


Figure 3.12 Histogram for electrical demand

When considering a power factor correction capacitor installation at an electrical plant, load conditions at light, average and full load must be considered [28].

The histograms show these categories, 500 – 750 (light load), 1000 – 1250 (average load) and 1750 – 2000 (full load).

To convert the histogram to a probability density function, let us assume that the probability of a demand less than 250kVA and the probability of a demand greater than 2000kVA are both 0.01, so that the probability between 250 and 2000kVA is 0.98.

Since the widths of the intervals are the same, the probability represented by each bar is proportionate to its height.

If the individual heights are divided by the sum of the bar heights and then multiplied by 0.98, we arrive at the following probability values, (rounded to the nearest 0.1 value):

$$\begin{aligned}
 \text{Sum of bar heights} &= 0.2 + 5 + 0.2 + 10 + 0.2 + 0.2 + 1 = 16.8 \\
 p(250 - 500) &= (0.2/16.8) 0.98 = 0.0117 \approx 0.0 \\
 p(500 - 750) &= (5.0/16.8) 0.98 = 0.2916 \approx 0.3 \\
 p(750 - 1000) &= (0.2/16.8) 0.98 = 0.0116 \approx 0.0 \\
 p(1000 - 1250) &= (10/16.8) 0.98 = 0.5833 \approx 0.6 \\
 p(1250 - 1500) &= (0.2/16.8) 0.98 = 0.0116 \approx 0.0 \\
 p(1500 - 1750) &= (0.2/16.8) 0.98 = 0.0116 \approx 0.0 \\
 p(1750 - 2000) &= (1.0/16.8) 0.98 = 0.0583 \approx 0.1 \\
 \hline
 &1.0
 \end{aligned}$$

The probability density function $f(s_d)$, for the 2000kVA electrical plant is:

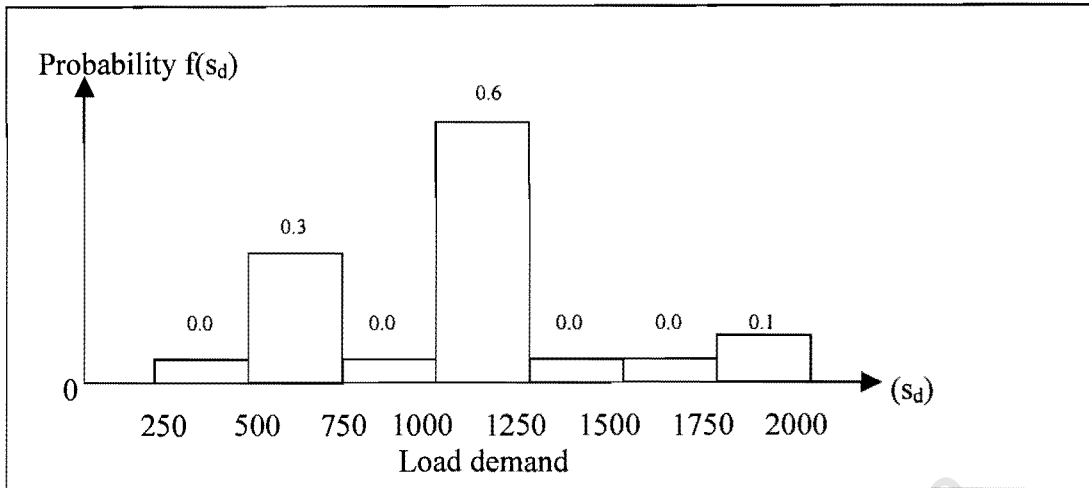


Figure 3.13 Probability function for 2000kVA plant.

Now, by cumulating the probability values, starting at the left of the probability density function, we can readily determine the cumulative distribution function (cdf) for the interval end points:

$$\begin{aligned}
 F(250) &= 0.0 \\
 F(500) &= 0.0 + 0.0 = 0.0 \\
 F(750) &= 0.0 + 0.3 = 0.3 \\
 F(1000) &= 0.3 + 0.0 = 0.3 \\
 F(1250) &= 0.3 + 0.6 = 0.9 \\
 F(1500) &= 0.9 + 0.0 = 0.9 \\
 F(1750) &= 0.9 + 0.0 = 0.9 \\
 F(2000) &= 0.9 + 0.1 = 1.0
 \end{aligned}$$

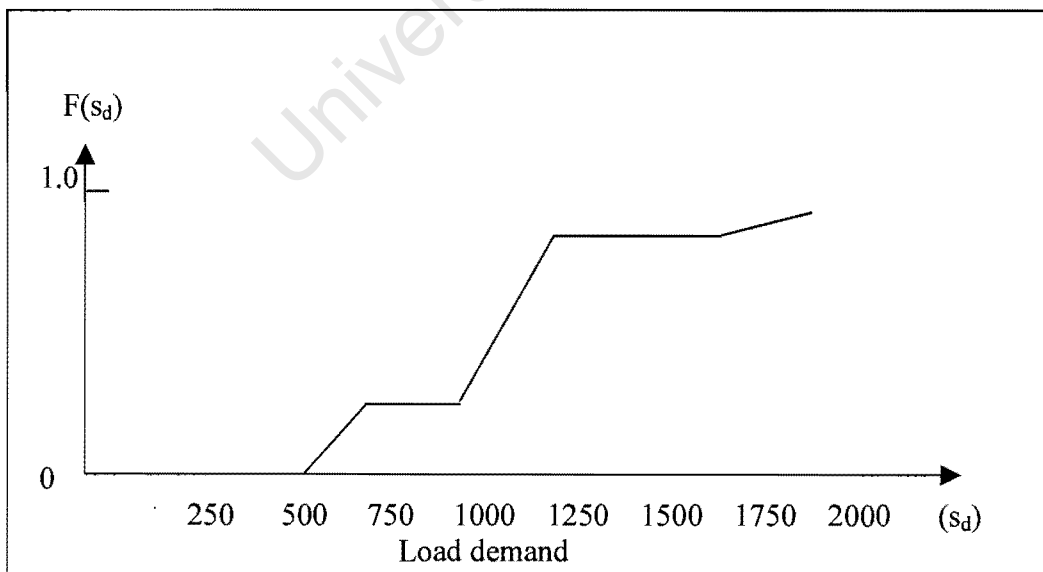


Figure 3.14 Cumulative distribution function for 2000kVA plant.

Given the probability density function of a continuous random variable (s), the cdf of (s), denoted by $F(s)$ is defined as the probability that (s) is less than or equal to some specific value of (s). Therefore, for the light load (500 – 750kVA), the probability is less than or equal to 0.3, for the average load (1000 – 1250kVA) it is less than or equal to 0.9 and for full load (1750 – 2000kVA) it is less than or equal to 1.0. Now applying equation (3.9):

$$\begin{aligned} p(500 \leq s \leq 750) &= F(750) - F(500) = 0.3 - 0.0 = 0.3 \text{ (30\%)} \\ p(1000 \leq s \leq 1250) &= F(1250) - F(1000) = 0.9 - 0.3 = 0.6 \text{ (60\%)} \\ p(1750 \leq s \leq 2000) &= F(2000) - F(1750) = 1.0 - 0.9 = 0.1 \text{ (10\%)} \end{aligned}$$

$$\underline{\underline{\sum p = 1.0}}$$

3.17 DECISION TABLE FOR MAKING A DECISION UNDER RISK

The main characteristic of decision-making under risk is that there are a number of events (states of nature), which might occur, but now reliable probabilities can be assigned to each of them.

Probability values can now be added to the decision table, which will now have the following general format:

DECISION TABLE FOR MAKING A DECISION UNDER RISK				
DECISION ALTERNATIVES	STATES OF NATURE			
	s_1	s_2	...	s_N
a_1	r_{11}	r_{12}	...	r_{1N}
a_2	r_{21}	r_{22}	...	r_{2N}
\vdots	\vdots	\vdots	\vdots	\vdots
a_n	r_{n1}	r_{n2}	...	r_{nN}
PROBABILITIES	p_1	p_2	...	p_N

Table 3.5 Decision Table for making a decision under risk.

3.17.1 DECISION MAKING USING THE EXPECTED MONETARY RULE

The traditional method for making a decision under risk is to calculate the expected value $E(X)$, for each decision alternative and then select the alternative with the best expected value.

This technique can only be used once probability estimates have been obtained from either the objective or subjective methods of assessment.

For the general case of a random variable (X) which has “ n ” different values (x_1, x_2, \dots, x_n) and which is associated with probabilities $p(x_1), p(x_2), \dots, p(x_n)$, $E(X)$ is defined as the weighted average of all possible values of (X) and is expressed as [27]:

$$E(X) = \sum_i x_i p(x_i) \quad (3.10)$$

In order to get the expected value, each possible value of the random variable is weighted by its probability of occurrence [product x_i times $p(x_i)$], then $E(X)$ is obtained by calculating the sum of all of the products, namely:

$$E(X) = x_1 p_1 + x_2 p_2 + \dots + x_n p_n \quad (3.11)$$

The expected value is not something that can occur in a single experiment. It is a long run average of repeated experiments. It is the weighted average of all the possible outcomes, each weighted by its probability.

For a given decision problem, a decision table lists states of nature against decision alternatives. If, for each combination, a monetary value (e.g. profit in dollars) is specified (obtained from calculations) as the outcome, the decision table is referred to as a payoff table.

The method for making a decision under risk when the outcome is a monetary value, makes use of the expected monetary value (EMV) rule to help choose the best decision alternative. That is, the rule helps to choose the decision alternative that gives the largest EMV. Using the results from the decision table, the EMV of alternative “n” in a decision problem is defined as [17]:

$$EMV_n = \sum_N r_{nN} p_N \quad (3.12)$$

where: r_{nN} = the payoff for alternative “n” under the N^{th} state of nature.
 p_N = the probability of the N^{th} state of nature.

3.17.2 EXAMPLE 3.5 – APPLICATION OF THE EMV RULE

To explain the application of the EMV rule, an example from reference [23] is adapted and used.

A company is to introduce a new computer-based toy to the children market. The company will have to build a new electrical plant to produce this product. Four different plant sizes are under consideration, small, moderate, large and very large. The appropriate size will depend on the level of demand for the product. However the level of demand is uncertain. It can be low, medium or high. The best information available was used to estimate the profits (\$) that the four different plant sizes would make.

For a small plant it forecasts a \$250000 profit with a low demand, a \$40000 loss with medium demand and no profit/loss (\$0) with high demand.

A moderate plant will give \$50000 loss with low demand, a \$350000 profit with medium demand and \$60000 profit with high demand.

CHAPTER 3

The large plant will give a \$100000 loss with a low demand, \$80000 profit with a medium demand and \$400000 profit with a high demand, respectively.

For the very large plant the prediction is \$120000 loss (low demand), \$75000 profit (medium demand) and a \$400000 profit (high demand), respectively.

The states of nature in this problem are mutually exclusive and collectively exhaustive events with a low, medium and high demand. There are thus a finite number of three states of nature and each is expressed in qualitative terms, namely:

- s_1 = low demand
- s_2 = medium demand
- s_3 = high demand

After polling a group of executives, industry officials and consumer panels about the profitability of the toy, the decision maker subjectively assigns probabilities to the states of nature as follows:

- $p(s_1) = 0.3$ (probability of low demand)
- $p(s_2) = 0.6$ (probability of medium demand)
- $p(s_3) = 0.1$ (probability of high demand)
- $\sum p(s_N) = 1.0$ (collectively exhaustive)

The situation is now one of risk rather than one of uncertainty.

There are four choices, therefore the decision alternatives are:

- a_1 = build a small plant
- a_2 = build a moderate plant
- a_3 = build a large plant
- a_4 = build a very large plant.

The decision objective is the highest profit. The outcome is therefore a monetary value. The outcomes would normally be derived from a quantitative model of the decision problem. To structure this otherwise unstructured decision problem, the following decision table for making a decision under risk is formulated:

DECISION TABLE – NEW ELECTRICAL PLANT			
DECISION ALTERNATIVES	STATES OF NATURE		
	s_1 = low demand	s_2 = medium demand	s_3 = high demand
a_1 = build a small plant	\$250000	-\$40000	\$0
a_2 = build a moderate plant	-\$50000	\$350000	\$60000
a_3 = build a large plant	-\$100000	\$80000	\$400000
a_4 = build a very large plant	-\$120000	\$75000	\$400000
PROBABILITIES	$p(s_1) = 0.3$	$p(s_2) = 0.6$	$p(s_3) = 0.1$

Table 3.6 Decision Table – new electrical plant.

At this stage the decision maker can check for “dominance”, that is, perform an initial screening to determine if some alternatives can be eliminated from the consideration.

When there is a low demand (s_1), the large plant (a_3) has a smaller loss than the very large plant (a_4). Likewise, in medium demand (s_2), the \$80000 profit from (a_3) is better than \$75000 from (a_4). For high demand (s_3), the profits are identical from (a_3) and (a_4). Thus (a_3) dominates (a_4) and can be eliminated. The “relevant payoff table” is:

RELEVANT DECISION TABLE – NEW ELECTRICAL PLANT			
DECISION ALTERNATIVES	STATES OF NATURE		
	s_1 = low demand	s_2 = medium demand	s_3 = high demand
a_1 = build a small plant	\$250000	-\$40000	\$0
a_2 = build a moderate plant	-\$50000	\$350000	\$60000
a_3 = build a large plant	-\$100000	\$80000	\$400000
PROBABILITIES	$p(s_1) = 0.3$	$p(s_2) = 0.6$	$p(s_3) = 0.1$

Table 3.7 Relevant Decision Table – new electrical plant

In a risk situation, anyone of the states of nature is possible. Thus the method used to make the decision should consider the outcomes associated with each state of nature. The criteria that best utilizes all available information in the decision table is the EMV and is used to make the decision. Using equation (3.12), the EMV’s are calculated as follows:

Small plant: $EMV_1 = (250000)(0.3) + (-40000)(0.6) + (0)(0.1) = \51000 profit.

Moderate plant: $EMV_2 = (-50000)(0.3) + (350000)(0.6) + (60000)(0.1) = \201000 profit

Large plant: $EMV_3 = (-100000)(0.3) + (80000)(0.6) + (400000)(0.1) = \58000 profit.

From the EMV results, the company should build a moderate plant (a_2) as this would give the highest profit.

The expected value is not the outcome that the company will get when selecting the recommended alternative. Actual profits depend upon demand and would be - \$50000, \$350000 or \$60000 for the three states of nature, respectively. Rather the EMV represents the average outcome that will result over the long run from continually repeating the decision alternative. This means if the company plans to build a large number of moderate plants, then the EMV of \$201000 is a good estimate of the average profit from such ventures. As subjective probabilities can affect the outcome, their assessment should be done in accordance with the approach given in figure 3.4.

3.17.3 LIMITATIONS OF THE EMV RULE

The application of the EMV rule for making decisions under risk has limitations [20].

1. The EMV rule focuses on only one attribute, namely money. It does not consider what each outcome is actually worth to a decision maker. It may not be appropriate if the decision is a one-time opportunity with substantial risks. If the company builds only one moderate plant and the demand is low, it will suffer a loss of \$50000 or make a profit of \$350000 if there is a medium demand or \$60000 profit for a high demand.

2. The EMV rule does not reflect real preferences of the decision maker. This becomes clear, as there are certain situations where the decision maker knowingly acts contrary to this rule. For example, people buy insurance for valuable articles. They understand that the EMV (as an investment) is negative. There will be a long run expected loss, yet people still purchase insurance. Likewise, many individuals gamble and play state lotteries, knowing their long run EMV will be negative. It goes about “attractiveness”. A prospect of a large one-time gain from a small payment is possible. Clearly, actual monetary values do not completely express “desirability” of decision makers. In fact most decision makers differ in their attractiveness for different amounts of money. Therefore, attractiveness is not a linear function whereas the EMV assumes a linear function for money. For example, a low paid worker who earns \$500 and receives a \$500 bonus finds the bonus very attractive, whereas a high paid worker who earns \$5000 and receives a bonus of \$500, may not find it as attractive as the lower paid worker.

Despite the limitation, the EMV rule for decision-making is widely used in practice. It has also been argued that it is appropriate to apply it to one-time decisions especially if a linear function is assumed for a decision maker.

3.18 UTILITY THEORY AS AN EVALUATION MODEL

Individuals do not always choose decisions based on EMV even when the payoff's and the probabilities for the states of nature are known.

A method based on an expected utility value is introduced. Its advantages over the EMV based method for making decisions under risk and its usefulness for making one-time decisions is discussed. This alternative approach is called the “Utility Theory” method and uses utility values, $U(V)$, which reflect the decision-maker's preference for each possible outcome in the decision table rather than pure monetary payoff's. The expected utility value is then used to make the decision. It uses $U(V)$ values rather than payoffs(monetary outcomes) [21].

Utility theory applies to all decision-making situations that individual persons might face while acting alone. At the outset, it is important to note that utility values can be constructed for [20], [24]:

- a. Monetary outcomes in a decision table.
- b. Outcomes that have a non-monetary value in a decision table.

For the moment, we will limit our discussion of utilities to monetary outcomes, which are conceptually more straightforward. Thereafter we will concentrate on utilities for non-monetary outcomes, as they are more applicable to engineering investigations.

3.18.1 DEFINITION OF THE TERM UTILITY

The term utility is defined as [27]:

“A subjective numerical measure of the value of an act to a decision maker when a particular event occurs”

3.18.2 UTILITY TABLE

The combination of an act and an event is an outcome. In utility theory numerical values (called utilities) are assigned to the various outcomes in a decision table. Utility theory is based on the assumption that every decision maker can translate each of the possible outcomes (monetary or non-monetary) in a decision table into a non-monetary numerical measure called a “utility”, (U), [17]. Once the outcomes have been replaced by their utilities, the table is now called a “utility table” and has the following general format:

UTILITY TABLE FOR MAKING A DECISION UNDER RISK				
DECISION ALTERNATIVES	STATES OF NATURE			
	s ₁	s ₂	...	s _N
a ₁	U ₁₁	U ₁₂	...	U _{1N}
a ₂	U ₂₁	U ₂₂	...	U _{2N}
⋮	⋮	⋮	⋮	⋮
a _n	U _{n1}	U _{n2}	...	U _{nN}
PROBABILITIES	p ₁	p ₂	...	p _N

Table 3.8 Utility Table for making a decision under risk

Where: U_{nN} = the utility measure assigned to an outcome (r_{nN}) in the decision table.

3.18.3 EXPECTED UTILITY VALUE

The utilities (outcomes) in the utility table can now be used to make a decision. This evaluation model is similar to the EMV model (equation 3.12), except for the introduction of the utility value (U_{nN}), which is unique to a particular decision maker. Therefore, instead of using the EMV rule, an “Expected Utility” value, (EU) is used to make the decision. The EU is then used to help choose the best decision alternative (largest EU value). Using the results from the utility table the EU of decision alternative “n” is defined as:

$$EU_n = \sum_N U_{nN} P_N \quad (3.13)$$

Here, all the utilities associated with a given alternative (a_n) and states of nature (s_N) are used to calculate the expected utility. The largest EU_n value is then the decision that needs to be implemented by the decision maker [23].

The use of the term expected utility could be misleading. It is used in utility theory, because the procedure for calculating expected utilities is arithmetically the same as that for calculating expected values. It does not refer to an average result that would be obtained over a long run nor does it mean a result that can be expected. In decision

theory, an expected utility is a single figure that is equivalent in preference to the uncertain situations [20].

3.18.4 AXIOMATIC THEORY OF UTILITY

Before we describe how to specify utility numbers, it is essential to mention further assumptions (axioms) upon which utility theory is based [20], [25], [27]:

AXIOM 1: (Preference ranking)

If outcome A is preferred to outcome B, then the utility for outcome A, $U(A)$, is greater than the utility for outcome B, $U(B)$, namely:

$$U(A) > U(B) \quad (3.14)$$

The axiom (established or accepted principle) is therefore that a decision maker can determine for any two outcomes, A and B, whether he prefers A to B, B to A or regards both equally ($A = B$).

AXIOM 2: (transitivity of preference)

Following on from axiom 1, if outcome B, $U(B)$ is preferred to outcome C, $U(C)$, then,

$$U(B) > U(C) \quad (3.15)$$

Then, if we make the assumption that preferences are transitive, outcome A, $U(A)$ would be preferred to outcome C, $U(C)$, since,

$$U(A) > U(B) > U(C) \quad (3.16)$$

This axiom is called transitivity of preference and reflects a decision-maker's consistency. It must be noted that the values $U(A)$, $U(B)$, etc, pertain only to a single decision maker who behaves consistently in accordance with his own beliefs.

The decision maker is therefore able to rank outcomes from best to worst [19].

AXIOM 3: (equal utility)

If a decision-maker is indifferent between two outcomes, then they have equal utility [24].

AXIOM 4: (continuity)

If there are three possible outcomes, $U(A)$, $U(B)$ and $U(C)$.

From axiom 2, it is an accepted principle that outcomes can be ranked from best to worst, e.g.:

$$U(A) > U(B) > U(C) \quad (3.17)$$

If $U(A)$ is the best outcome and $U(C)$ is the worst outcome, then $U(B)$ is called an in-between outcome.

This axiom states that if the decision-maker is indifferent between outcome B, $U(B)$, which is sure outcome and the outcome of a lottery in which the decision-maker receives outcome A, $U(A)$ with a probability of (p) and outcome C, $U(C)$ with a probability of $(1-p)$, then the assumption is that [23], [27]:

$$U(B) = \text{Expected Utility of the lottery ticket.}$$

That is:

$$U(B) = pU(A) + (1-p)U(C) \quad (3.18)$$

where: $pU(A) + (1-p)U(C)$ is the expected utility of the lottery ticket and is used to calculate the utility of the sure outcome.

$$\text{Therefore: } U(\text{sure outcome}) = pU(\text{best outcome}) + (1-p)(\text{worst outcome}) \quad (3.19)$$

The basis of this axiom and utility theory in general is that a decision-maker will select a course of action, which will attempt to maximize the expected utility. Alternatively, the continuity axiom states that there must be some value of (p) at which the decision-maker will be indifferent between the sure outcome and the outcome of the lottery [20].

The expected utility of the lottery ticket is not to be confused with the expected utility used in equation 3.13 to help choose the decision alternative “n” and make a decision under risk.

3.18.5 VARIABLE PROBABILITY METHOD FOR DETERMINING UTILITY VALUES

The elicitation procedure for determining utility values for in-between outcomes is based on axiom 4 and is called the variable probability method. Alternative names for this method are: probability equivalence approach, lottery indifference probability method, a reference lottery or an equivalent lottery [17] to [27].

An example from reference [17] is adapted and used in the next section to explain the procedure.

3.18.6 EXAMPLE 3.6 – PURCHASING AN ELECTRICAL PLANT

Suppose, we could buy either of the two electrical manufacturing plants given in the following decision table:

DECISION TABLE : PURCHASING AN ELECTRICAL PLANT		
DECISION ALTERNATIVES	STATES OF NATURE	
	1	2
Plant A	\$140000	-\$20000
Plant B	\$60000	\$30000
PROBABILITIES	0.5	0.5

Table 3.9 Decision Table for purchasing an electrical plant

Nb! The \$140000 indicates a profit and -\$20000 indicates a loss.

Let $U(x)$ represent utility associated with a payoff of \$x.

To begin with we represent utilities on a scale from 0 to 1. Utilities of 0 and 1 are assigned to the worst and best outcomes respectively, and appear in the decision table, namely:

$$\begin{aligned} U(-20000) &= 0 \text{ (worst outcome)} \\ U(140000) &= 1 \text{ (best outcome)} \end{aligned}$$

We now need to find utility values for in-between outcomes. Lets say we wish to find a utility for \$60000. To do this, we must identify the probability p at which the decision-maker is indifferent between the following two alternatives:

- Alternative 1: Receive \$60000 as a sure outcome
 Alternative 2: Receive \$140000 (best outcome) with probability p
 and lose \$20000 (worst outcome) with probability $(1-p)$

If $p = 0$, alternative 1 would be chosen, as the decision-maker would prefer to receive a payoff of \$60000 rather than lose \$20000.

If $p = 1$, alternative 2 would be chosen as he would prefer \$140000 rather than \$60000.

Therefore, as p increases from 0 to 1, a point p will be reached where the decision-maker is indifferent between the two alternatives. This probability point is called the probability indifference value (p_{ind}).

For the purpose of this explanation, a $p_{ind} = 0.8$ is assumed for this in-between value. Equation 3.19, is now used to calculate the utility value,

$$U(60000) = p_{ind} U(140000) + (1-p_{ind}) U(20000)$$

but, $U(140000) = 1$ and $U(20000) = 0$

$$U(60000) = (1)p_{\text{ind}} + 0(1-p_{\text{ind}}) = p_{\text{ind}} = 0.8$$

Next we need to find the utility value for the in-between outcome of \$30000. Once again we must identify the p_{ind} value applicable to the following:

- Alternative 1: Receive \$30000 as a sure outcome
- Alternative 2: Receive \$140000 with a probability p and lose \$20000 with a probability $(1-p)$.

For explanation purposes, and as the payoff is reduced we assume that $p_{\text{ind}} = 0.6$. The utility associated with this payoff is then:

$$\begin{aligned} U(30000) &= p_{\text{ind}} U(140000) + (1-p_{\text{ind}}) U(20000) \\ &= p_{\text{ind}} = 0.6 \end{aligned}$$

Again, the in-between utility is equivalent to the decision-maker's p_{ind} .

Thus, when a scale of 0 to 1 is used p_{ind} always corresponds to the decision-maker's utility for the outcome listed in alternative 1.

The derived utility values are then used to draw up the utility table for this decision problem.

UTILITY: TABLE PURCHASING AN ELECTRICAL PLANT		
DECISION ALTERNATIVES	STATES OF NATURE	
	1	2
Plant A	1	0
Plant B	0.8	0.6
PROBABILITIES	0.5	0.5

Table 3.10 Utility Table for purchasing an electrical plant.

The utilities in the utility table can now be used to make a decision on which of two electrical plants should be purchased.

Equation 3.13 is used to calculate the expected utility values for the two decision alternatives, plant A or plant B.

$$EU_{(\text{plant A})} = (0.5)(1) + (0.5)(0) = 0.5$$

$$EU_{(\text{plant B})} = (0.5)(0.8) + (0.5)(0.6) = 0.7 \text{ (largest).}$$

The largest EU is plant B, therefore the decision is to purchase plant B.

Now using the EMV rule (equation 3.12), we get:

$$EMV_{(\text{plant A})} = (0.5)(140000) + (0.5)(20000) = \$60000 \text{ (largest).}$$

$$EMV_{(\text{plant B})} = (0.5)(60000) + (0.5)(30000) = \$45000$$

Therefore, according to the EMV rule, plant A should be purchased and not B. Although, A would generate the highest EMV over a long run, the purchase may not have the resources to withstand a potential loss of \$20000 over the short run. With B, the purchase can be sure of making at least \$30000, although B's EMV over the long run might not be as great as A.

Thus by using utility theory, a decision-maker can identify the decision alternative that is more attractive, given their preferences about risk and profit.

The utility function of the decision maker can be plotted. Using tables 3.9 and 3.10, the utility function for this example is curve A.

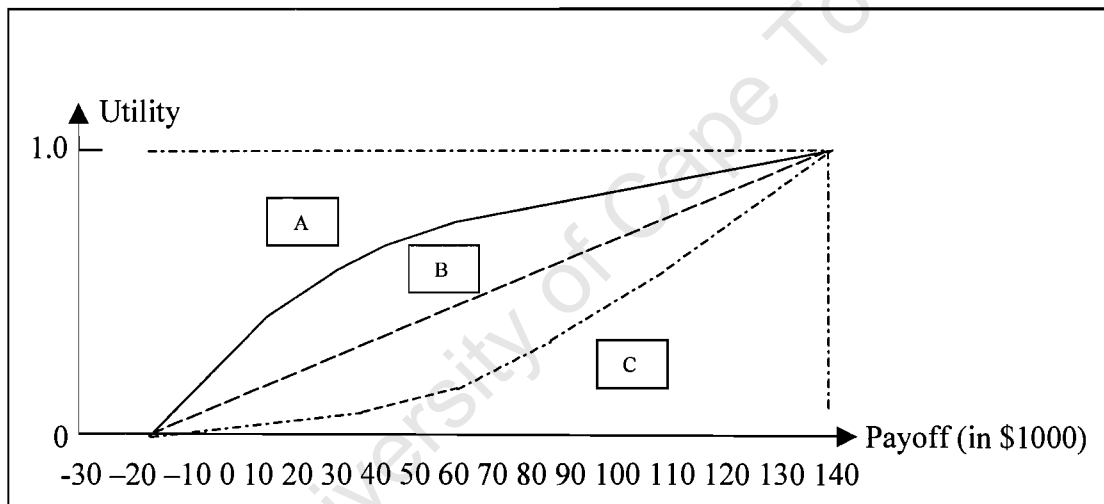


Figure 3.15 Utility function – decision-maker electrical plants

Different decision-makers have different preferences towards risk and return. Those who are “risk neutral” have a linear utility function similar to curve B. However, those who are risk averse (risk avoiders) generally have a function similar to curve A. Others look for risk and generally have functions similar to curve C and are categorized as risk seekers. Risk averse and risk seekers typically have concave and convex utility functions, respectively [17].

In practice, a utility function is found empirically by personally interviewing the decision-maker. Where the decision-maker is oneself, the same interviewing procedure is applicable. A series of questions based on the hypothetical win-lose gamble are asked until the decision-maker is indifferent (p_{ind}). An example of a typical interview for constructing a utility function can be seen in reference [25].

In general, utility curves can assume virtually any form as they depend on the preferences of a decision-maker.

3.19 UTILITY VALUES FOR NON-MONETARY OUTCOMES

The concept of utility is even more important when decision problems have outcomes, which do not have monetary values. For instance, without a numerically valued payoff the EMV rule cannot be applied and used for decision-making.

An example from reference [24] is adapted and used to explain how utilities are assigned to non-monetary outcomes and is shown in Appendix 9, section 9.2. The variable probability method as used in Example 3.6 is applied to assign utility values to the outcomes in a decision table.

The example in Appendix 9, section 9.2 shows that although no specific monetary payoff values are available, the decision-maker can still make an intelligent choice by using utility theory.

The derived utility values can be used to derive the utility function for the decision-maker. As the best outcome has a utility of 1 and the worst a utility of 0, the utility function graph will be an ascending concave curve and this shows that the decision-maker is risk averse.

As stated in the last paragraph of chapter 3, section 3.18.6, utility function graphs can take on any form. For example, where the outcome is a non-monetary value, the highest value outcome can be the worst outcome and the lowest value the best outcome, resulting in a utility function graph with a descending concave curve instead of an ascending curve as in figure 3.15 [20].

3.20 PRACTICAL USEFULNESS OF UTILITY THEORY

We have seen that utility theory can provide guidance on how to choose between alternative courses of action. Is it worth taking the trouble of asking the decision-maker a series of questions based on the variable probability method [20]?

1. In decision problems that do not involve a high level of risk, utility theory has been found to play a valuable role as long as the decision-maker is familiar with the concept of probability and has the time and patience to devote the required effort and thought to the series of questions being asked during the elicitation process.
2. Utilities are not perfect measures but are found to be a useful tool for gaining greater understanding of the decision problem.

3. The utility evaluation model is similar to the expected monetary value model except for the introduction of the utility variable (U). Some persons would object to use the utility model, as it is not an objective model like the expected monetary value model because it incorporates subjective judgments. However, it must be stressed, the proper criterion for choosing an evaluation model is how well it captures the true preferences of the decision-maker. Since the expected utility evaluation model explicitly includes these preferences, it is considered to be superior to the expected monetary value model [29].
4. The utility evaluation model is extremely valuable when outcomes in decision tables are non-monetary values expressed as percentages. As many electrical engineering models have outcomes expressed in percentages, utility theory can be extended to solve such problems.

3.21 SUMMARY

The importance of the subjective method for eliciting probabilities for the states of nature in a decision problem is highlighted. In particular, the method of relative heights is introduced together with a worked example to demonstrate its usefulness to the decision theory process when objective probabilities are not obtainable. The general format for a decision and utility table are developed for making decisions under risk. The importance of utility theory as an evaluation model and the value of the variable probability model for eliciting utilities for monetary and non-monetary outcomes of a decision as well as the application of the expected utility rule, is demonstrated by means of worked examples. The practical usefulness of utility theory for making decisions under risk is demonstrated. The chapter also prepares the reader for the application of decision theory to the making of a mitigation decision on harmonic resonance.

CHAPTER 4

DECISION THEORY PROCESS FOR MAKING A MITIGATION DECISION ON HARMONIC RESONANCE

In this chapter a new process based on decision theory is introduced. A three-stage process for making a harmonic resonance mitigation decision on pf correction capacitors in an end-user plant is developed. Two new indexes are developed to assist in making the decision. The first index assesses the severity of harmonic resonance and the second index is used to make a mitigation decision. In stage 1 of the process a quantitative decision model is developed. States of nature are identified in terms of probabilities so that a decision is made in the risk zone making the model deterministic. A resonance frequency band is implemented together with a 2-controllable input approach for identifying decision alternatives. The harmonic resonance severity index is introduced as the objective function to solve the decision problem. A new decision table is developed to structure and represent the harmonic resonance problem. In stage 2, a new application for utility theory and the variable probability method is developed for making a decision on the most desirable capacitor size (decision 1). In stage 3, the new mitigation index is applied to assess if mitigation is needed for the chosen capacitor (decision 2). A template for making a mitigation decision on harmonic resonance is developed. This chapter serves as theoretical background and lays a foundation for the case studies, which follow, in the next chapters to demonstrate the usefulness of this newly developed decision theory process.

4.1 BACKGROUND

The use of capacitors for power factor correction is a common practice in the power industry. With the proliferation of harmonic producing loads and the increased awareness of harmonic effects, the possibility of capacitor-system resonance at harmonic frequencies has become a concern for end-users and capacitor installers.

The problem with capacitors is that they are known to be sensitive to over-voltages and are sinks for harmonic currents. Traditionally, if there is an existence of harmonic resonance it is assumed to be severe and the practice is to mitigate the problem. In many cases this process is at the expense of the end-user.

However, the existence of harmonic resonance does not necessarily imply that a capacitor problem would occur, as the severity of resonance may not be sufficient to cause damage [3].

As the term severity of harmonic resonance is not defined in literature, there is a need to formulate its mathematical meaning at key harmonic frequencies. Therefore an index is needed to quantify the level of severity of harmonic resonance.

How to make a “decision” on the size of capacitor bank to be installed in an end-user plant based on the severity of harmonic resonance is not disclosed in literature. There is thus a need to formulate a decision process for making a decision on the size of a capacitor to be installed.

Once a decision has been made on the size of the capacitor bank, there is a further need to evaluate the situation and to determine if a mitigation solution is needed or not.

Also, even though a high severity of harmonic resonance may be found in the end-user system, it may not necessarily damage the capacitor bank. Therefore, there is still a further need to put in place a second index to quantify the level at which mitigation is needed. Such an index should not only take into account the decision on the size of the capacitor to be installed but also the severity of resonance.

Most importantly of all, as there is a decision to be made, there is a need for a decision theory process to guide the decision-maker (capacitor installers).

4.2 TWO-STAGE DECISION THEORY PROCESS

Management scientists have developed a rational methodology for conceptualizing, analyzing and solving decision problems, called the decision theory process and this has been reviewed in chapter 3.

Figure 3.2 in chapter 3 is a block diagram representation of this scientifically developed process. It is found that the process is comprised of two main stages, a quantitative model building stage and a decision-making stage. There are three zones for making decisions, uncertainty, risk and certainty. When probabilities are assigned to each state of nature the decision moves from uncertainty to the risk zone and the model is called a deterministic model. For reasons given in chapter 3 (section 3.13.1), the most relevant zone for making decisions is the risk zone. If the outcome measure in a developed decision table is a non-monetary value, then utility theory is used to make the decision rather than the EMV rule.

4.3 THREE-STAGE PROCESS FOR MAKING A MITIGATION DECISION ON HARMONIC RESONANCE

The focus of the research is the severity of resonance. As the severity of harmonic resonance is to be formulated as a percentage (indexes are usually percentages), the outcomes in the developed decision table will be non-monetary values.

For this reason, it was necessary to research decision theory and to find a methodology and a solution for making a decision when an outcome is a non-monetary value. Through research it was found that management scientists had identified this problem and therefore had developed a second part to utility theory to address this issue. On further investigation it was found that the decision theory process and its link to utility theory and non-monetary outcomes had not been applied anywhere to electrical power systems and definitely not to problems on severity of harmonic resonance.

CHAPTER 4

From what was learnt, a quantitative model and a decision table are needed to represent a decision problem on the severity of harmonic resonance. As resonance depends on plant loading, states of nature need to be formulated and probabilities assigned to them. The situation is thus going to be one of decision making under risk and as the outcomes are to be non-monetary values, utility theory will be required as a tool to choose the size of the capacitor bank to be installed. Once the size of the bank has been chosen its severity of harmonic resonance will also be known. After this, a decision has to be taken as to whether or not mitigation is needed, if so, a solution is proposed.

From what has been said, it was deduced that a two-stage decision theory process was not adequate and that a three-stage process was needed.

Using decision theory, the following three-stage process has been developed and is shown in figure 4.1 [6], [7].

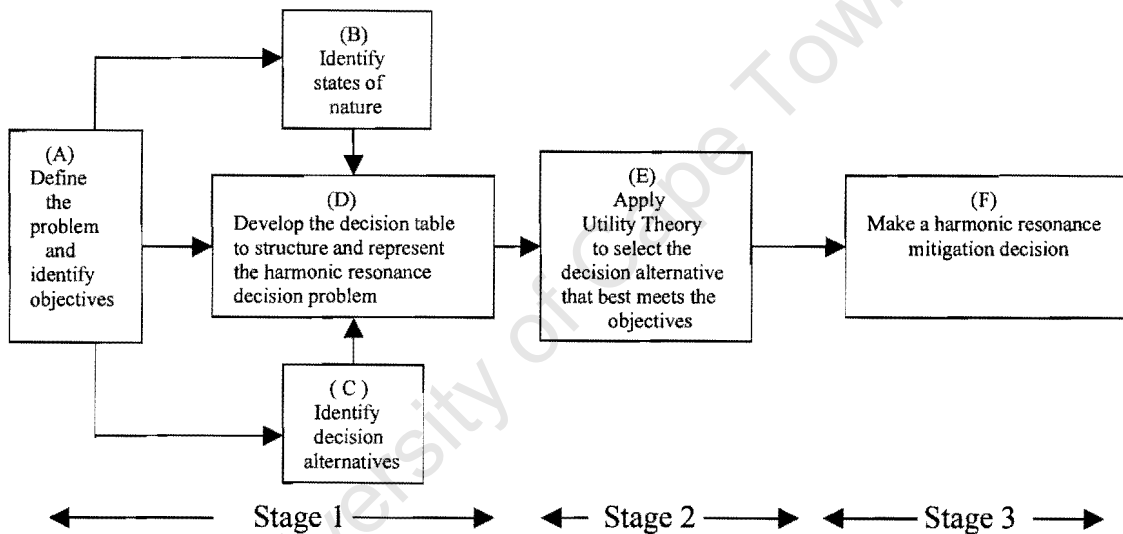


Figure 4.1 Decision theory process for making a mitigation decision on harmonic resonance

The main stages of the process are:

STAGE 1: A quantitative model building stage and its representation as a decision table.

STAGE 2: A decision-making stage

STAGE 3: Mitigation decision stage.

What follows is an explanation of the components of this new application of decision theory to harmonic resonance. The basic principles relevant to this research topic have been discussed in chapters 2 and 3.

4.4 DEFINE THE PROBLEM AND OBJECTIVES (BLOCK A)

The first component in the process is the definition of the decision problem. An example of a typical decision problem is given in example 3.5.

Building a quantitative model forces a decision-maker to determine what fundamental questions to ask and where to focus attention.

4.4.1 DECISION SCENARIO

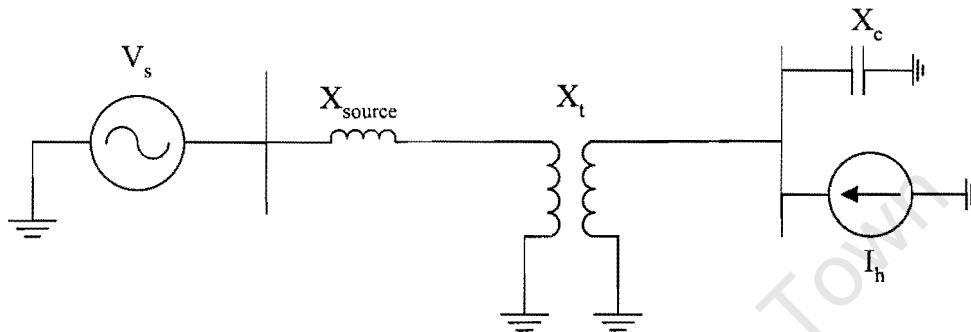


Figure 4.2 System with a potential severity of harmonic resonance

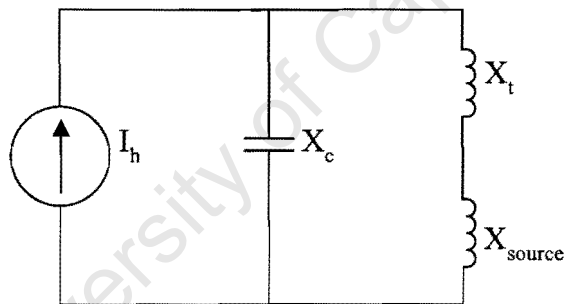


Figure 4.3 Equivalent circuit

Figures 4.2 and 4.3 represent a typical end-user system in which only one capacitor is present and only one resonant point occurs per capacitor size. The network shown is not valid for a system comprising of an interconnection of capacitors and reactors. This simple system is used to help with the development of the new decision theory process for making a mitigation decision on harmonic resonance in a power system [11].

An end-user (consumer) wants to introduce a pf correction capacitor bank to his plant to improve his low dpf. He is aware that his plant has a harmonic source (drive) and together with the capacitor bank to be installed, that resonance could occur at one of the key harmonic frequencies. He is also aware that the resonance could be severe and that there may be a need for a mitigation device to be installed to prevent damage to his capacitor bank. A further concern is that the value of power demand drawn by the plant is uncertain and that this can impact on the severity of harmonic resonance.

4.4.2 PROBLEM (BLOCK A)

The problem is will the harmonic resonance be severe enough to result in damage to the pf correction capacitors?

4.4.3 OBJECTIVES (BLOCK A)

The next component of the process is to identify the objectives.

Decisions are made to achieve certain objectives; therefore decision models include an explicit performance measure that gauges the attainment of that objective. Therefore, one variable must be chosen which acts as a measure of how good the outcome is with respect to the objective. It must take on a numeric value and be able to be judged good or bad. The variable chosen must be meaningful so that the model leads to a meaningful solution and give insights, which are largely qualitative.

Based on the problem statement above, the objectives to be met are:

- a. Determine the severity of harmonic resonance caused by the installation of pf correction capacitors at an end-user plant at key harmonic frequencies for a given range of power demand (steady-state) operating conditions. That is, make a decision between different sizes of capacitors, taking into account the preferences (utilities) of the decision-maker.
- b. Furthermore, if a high level of severity of harmonic resonance is found, make a decision if mitigation is needed or not, so as to prevent damage to the capacitor installed in the plant.

4.5 IDENTIFICATION OF STATES OF NATURE (BLOCK B)

The next step in the process is to identify the conditions that are expected to occur. Such events are usually beyond the decision-maker's control (uncontrollable inputs) and are called the states of nature (s_1, s_2, \dots, s_N).

In many situations, there are an unlimited number of possible events, particularly when conditions are expressed in quantitative terms. There could be an infinite number of events. In other cases, the problem format will limit (constrain) the number of states of nature to a finite range of values and the decision-maker may prefer to describe the range by a few qualitative categories.

It is also advisable to define the states of nature in a way that precludes two (or more) events from occurring simultaneously. The occurrence of one state will then exclude all others. Such events are said to be mutually exclusive and one of them can only occur in a single trial of an experiment. It is also necessary to make certain that one of the events will occur. Thus, states of nature must also be collectively exhaustive [Chapter 3, section 3.9 (d) & (e)].

CHAPTER 4

It is therefore essential to identify conditions that are expected to occur. As power demand is the state of nature in our decision scenario, let the power demand conditions that occur in an end-user's plant (S_E) be represented by " s_N ". Suppose, there are $N \geq 1$ states of nature, then in general:

$$S_E(s_N) = [P_E(s_N)^2 + Q_E(s_N)^2]^{1/2} \quad N = 1, 2, 3, \dots, n \quad (4.1)$$

where: S_E , P_E and Q_E are the apparent, real and reactive powers for the end-user plant at fundamental frequency.

With no pf capacitor installed and if only the fundamental frequency is present, then the term displacement power factor (dpf) is used to describe the relationship between P_E and S_E (see chapter 2, equation 2.56), namely:

$$\text{dpf} = \cos \phi_E(s_N) = \frac{P_E(s_N)}{S_E(s_N)} \quad (4.2)$$

When sizing capacitors for pf correction, the fundamental frequency is used as capacitors can only correct $Q_E(\omega_1)$. At ω_1 they are sources of reactive power but at harmonic frequencies they are sinks [11].

In our scenario, it is usual to constrain the number of states of nature to a finite range and identify them by qualitative categories.

4.5.1 RECOMMENDED GUIDELINE

The recommended guideline and the first step when designing any practical pf correction scheme is to obtain loading details for the three most common categories found in end-user plants, namely minimum, mean and full load operating categories [28].

Therefore, the recommendation is to use a finite number of states of nature and express each of them in qualitative terms. The states of nature (steady-state) are therefore constrained, mutually exclusive and collectively exhaustive and can be identified as follows:

$$\begin{aligned} s_1 &= \text{minimum demand} \\ s_2 &= \text{mean demand} \\ s_3 &= \text{full demand} \end{aligned}$$

If " n " represents the variation in demand, these qualitative insights can be defined in terms of the full load category, namely:

$$S_N = n \times s_3 \quad 0 < n \leq 1 \quad (4.3)$$

It is also a strong recommendation of decision analysis to make decisions in the risk zone [22]. The concepts and principles relevant to making decisions under risk are covered in chapter 3, section 3.13.

To obtain a deterministic model it is necessary to obtain and assign probabilities for the plant (S_E) at minimum, mean and full load state of nature (s_N) categories.

The probabilities can be objectively or subjectively found by the objective and subjective methods described in chapter 3 (section 3.15.1 and 3.15.2, respectively). If the plant physically exists, probabilities (a posteriori) can be obtained from discrete or continuous probability distributions (chapter 3, sections 3.15 and 3.16, respectively). Usually recordings (e.g., 7 days) are conducted at the site, histograms (chapter 3, figures 3.5, 3.8 and 3.9) are created yielding occurrence information. Then, from the probability distributions for the states of nature, probabilities can be ascertained. Examples are given in chapter 3 on how to ascertain probability values (examples 3.1 and 3.3 respectively).

If the plant does not exist (a priori), a unique event and is in the process of planning, the subjective assessment method of “relative heights” (chapter 3, section 3.15.2) can be applied to elicit probabilities about the states of nature from derived discrete or continuous probability distributions. Examples are given in chapter 3 on how to obtain probability values (examples 3.2 and 3.4).

To move from uncertainty to the risk zone, the elicited probabilities must be assigned to the states of nature (s_N).

Let, $p(s_N)$ represent the probabilities assigned to the states of nature (s_N), thus,

$$0 \leq p(s_N) \leq 1 \qquad N = 1, 2, \dots, n \qquad (4.4)$$

4.6 IDENTIFICATION OF DECISION ALTERNATIVES (BLOCK C)

Making good decisions is rarely an easy task. Decision-makers are often faced with numerous possible courses of action. Evaluating these alternatives and choosing the best course of action represents the essence of decision analysis.

A decision alternative is therefore a course of action to solve a problem. These variables are the factors that influence the models outcome and are controlled by the decision-maker. That is, the decision-maker can change and manipulate these variables at will.

4.6.1 CHARACTERISTICS OF DECISION ALTERNATIVES

Identifying decision alternatives is part of the quantitative model building stage. The main characteristics of decision alternatives are summarized in chapter 3 [section 3.9 (a) to (g)].

An important characteristic is that the decision problem (scenario) will permit the number of decision alternatives within a set (a_1, a_2, \dots, a_n) to be constrained so that they are meaningful (feasible). This means the initial number of inputs (a_1 to a_n) could be limited to a narrow range and in turn could still be limited even further to a minimum number of

two alternatives within the narrow range (e.g., a_1 and a_2 within a narrow range a_1, a_2, \dots, a_n). A decision must involve at least two alternatives (e.g., a_1 and a_2), otherwise there is no decision to be made.

A further characteristic is that only those alternatives, which the decision-maker wants, needs to be included. The decision alternatives chosen within a set must however be mutually exclusive.

4.6.2 RESONANCE FREQUENCY BAND APPROACH AND THE 2-CONTROLLABLE INPUT DECISION MODEL

The following methodology is introduced for identifying decision alternatives (controllable inputs).

STEP 1:

For the given end-user plant (e.g., figure 4.2), derive for the full load demand state of nature (s_3), its fundamental frequency power triangle and displacement power factor $\cos \phi_E$ when no capacitor is installed.

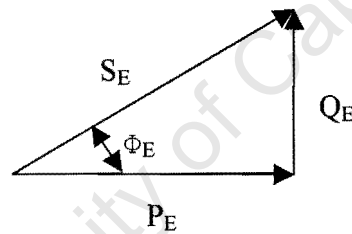


Figure 4.4 Power triangle for end-user plant

As the size of the capacitor (CAP1) is a factor that influences harmonic resonance and over which the decision-maker has control, let Q_C represent the kvar value for CAP1 to be installed in the plant. As Q_C is controllable, let $\%Q_C$ represent step settings for CAP1. The tendency today in industry is towards standardization of capacitor steps rather than optimization [31].

The power diagram for full load demand of the plant, after CAP1 is installed is illustrated in figure 4.5:

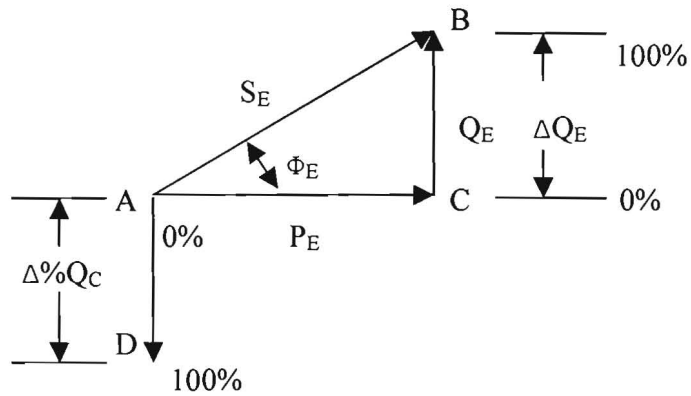


Figure 4.5 Power triangle with capacitor installed

where: ΔABC = state of nature (s_3).

$$AD = \text{Controllable input } \Delta\%Q_C$$

A problem with capacitors is that they are sinks for harmonics and are usually the first component to be damaged by harmonic resonance. A possible solution to this problem is to alter the frequency response characteristic of the system by choosing the most appropriate size of capacitor to control the severity of harmonic resonance [2].

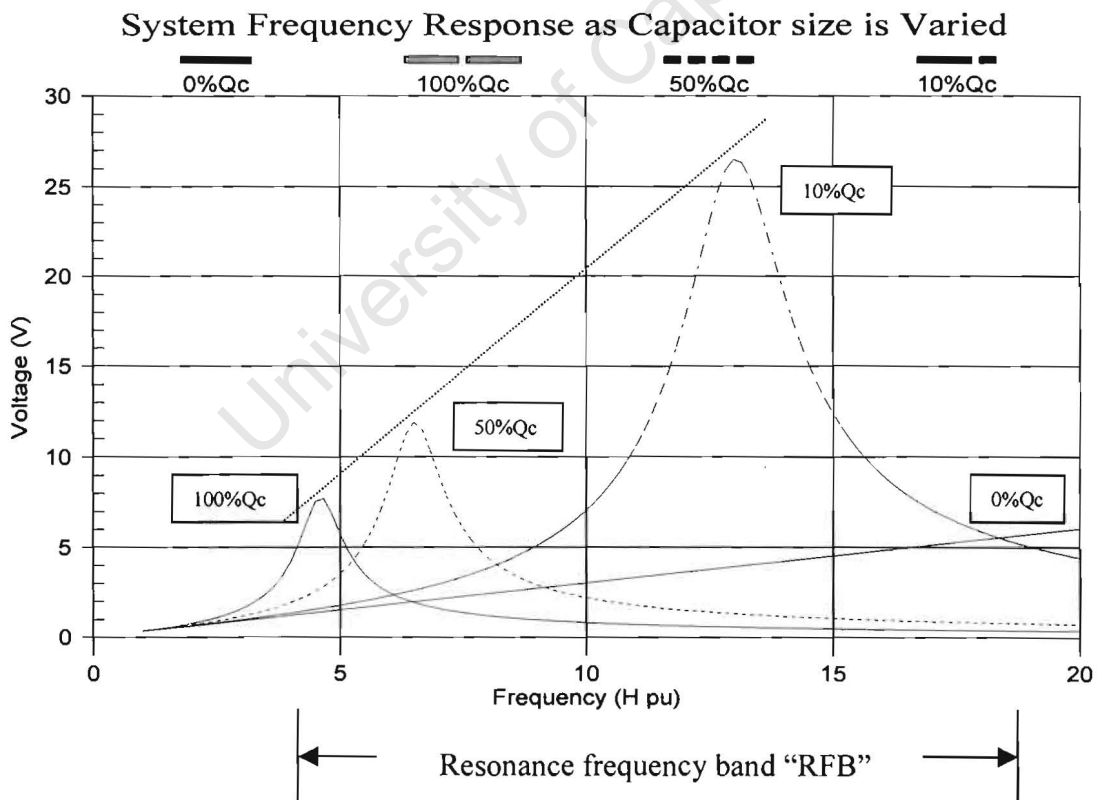


Figure 4.6 System frequency response as capacitor size is varied.

CHAPTER 4

Figure 4.6 shows typical systems frequency response as the capacitor size (%Q_C) is varied [11], [30]. The Electrotek Harmonic Analysis Software Package [37] is used in this thesis for all the simulation studies. This software package plots the frequency in per-unit (Hpu) against voltage (V), thus 5, 10, etc mean multiples of the fundamental frequency (h).

Appendix 2 shows an example of the models and parameters that would be typically used to investigate an end-user plant and to obtain the plots like in figure 4.6. Then, using the “Harmonic Scan Program” of the package, plot the resonance curve for the selected step settings to visualize the effects of capacitor size change on resonant points as the size is controlled from 0%Q_C (dpf = cos φ_E = 0, no capacitor) to 100%Q_C (dpf = cos φ_E = 1).

Let, %Q_C[fr(h)] represent the capacitor size to tune the network to resonate at fr(h). From the plots, identify the outer limits 0%Q_C[fr(h)] to 100%Q_C[fr(h)], then use these values to identify the “Resonance Frequency Band (RFB)” on the plot diagram of figure 4.6.

Also, let,

$$100\%Q_C = \%Q_{C(MAX)} \text{ (unity dpf)} \quad (4.5)$$

The range for sizing the capacitor is thus:

$$\text{Range 1} = 0\%Q_C < \%Q_C \leq \%Q_{C(MAX)} \quad (4.6)$$

The corresponding dpf range would be:

$$\cos \phi_E(0\%Q_C) < \cos \phi_E \leq 1 (\%Q_{C(MAX)}) \quad (4.7)$$

The driving point impedance (Z_D) values at fr(h) equal the driving point voltages (V_D) at fr(h), as scan studies usually use a constant current injection [I_(inject)] of 1∠0° A, thus:

$$V_D = I_{(inject)} Z_D \quad (4.8)$$

$$V_D = Z_D \quad (4.9)$$

The apex values of the resonant curves for the %Q_C capacitor sizes used within the RFB fall on a straight line when systems are simple as the one that is assumed for the scenario. The equation for the straight line is:

$$V_D = \frac{dV_D}{dfr(h)} fr(h) + K \quad (4.10)$$

where: “K” is a constant.

This equation can be used for calculating intermediate resonant points within the RFB.

STEP 2:

From the literature study conducted on decision tables, it was found that in business applications, only one set of decision alternatives is used and this seems to be the norm. (see chapter 3, table 3.1) [18].

However, when considering the harmonic resonance scenario, the following question was asked:

“Does the decision maker have control over any other parameter besides capacitor size that is relevant to the objectives set for the decision problem posed and which can be influenced by the states of nature and also influence the outcome (severity of harmonic resonance)?”

As the outcome is severity of harmonic resonance, it was found that not only does capacitor size influence the severity but that harmonics also play a major role.

The next question that was posed is:

“Are harmonics in the system also a controllable input?”

It was concluded that characteristic harmonic frequencies (hch) injected by a harmonic source (drive) in a system are known, and are therefore also a controllable input and a “set” for them would need to be defined and this set would be in addition to the first set, which was the variation of capacitor size.

In a symmetrical balanced steady-state system, the harmonic components for harmonic sources (e.g., drives) are injecting currents at “hch” and are related to the pulse number (p) of the drive as follows [33], [44], [45]:

$$hch(k) = pk \pm 1 \quad k = 1, 2, 3, \dots, q \quad (4.11)$$

A common contributor to power system harmonic distortion is the 6-pulse drive (Chapter 2, section 2.4.6). Thus for a 6-pulse drive:

$$hch(k) = 6k \pm 1 \quad k = 1, 2, 3, \dots, q \quad (4.12)$$

For investigating resonance, two controllable inputs are identified, %Q_C and “hch”, respectively.

Controllable Input 1

Let, “a_n” be the representation for the controllable input 1, alternatives. Here, “a” represents the %Q_C parameter and “n” the number to choose from (n ≥ 1), then,

$$a_1 = \% Q_{C(a_1)}, a_2 = \% Q_{C(a_2)}, \dots, a_n = \% Q_{C(a_n)}$$

The controllable input 1 set of alternatives is (a_1, a_2, \dots, a_n) .

Controllable Input 2

Let “ a_{nm} ” represent the controllable input 2, alternatives. Then “ a_{nm} ” represents the m^{th} characteristic harmonic coinciding to n^{th} controllable input 1 (a_n), alternative.

Therefore, for example, for a 6-pulse drive, the characteristic harmonics (hch) are:

$$a_{nm} = hch(k) = 6k \pm 1 \quad k = 1, 2, 3, \dots, q \quad (4.13)$$

For a given drive rating, “ S_{drive} ”, the magnitudes of the characteristic harmonic currents can be established by:

- Formula (see equation 2.62) – an ideal converter.
- Measurements, if plant (a posterior) exists.
- Published data [11].

The spectrum of a drive is therefore under the control of the decision-maker.

Also, the following observations can be deduced from equation 4.13, namely: (see chapter 2, equation 2.60).

- The absence of triplen harmonics.
- The harmonics of order $6k+1$ are of positive sequence. (h^+).
- The harmonics of order $6k-1$ are of negative sequence (h^-).

Therefore, in terms of negative (h^-) and positive (h^+) sequence symmetrical components.

$$h^-(k) = 6k - 1 \quad k = 1, 2, 3, \dots, q \quad (4.14)$$

$$h^+(k) = 6k + 1 \quad k = 1, 2, 3, \dots, q \quad (4.15)$$

thus, the controllable input 2 set (per controllable input 1) is $(a_{11}, a_{12}, \dots, a_{nm})$, where:

- a_{11} - represents the negative sequence component of the current for $k = 1$ (5^{th} harmonic), coinciding to controllable input 1 (a_1) alternative.
- a_{12} - represents the positive sequence component of the current for $k = 1$ (7^{th} harmonic), coinciding to controllable input 1 (a_1) alternative.

a_{13} - 11th harmonic ($k=2$, negative sequence – coincide to a_1).

a_{14} - 13th harmonic ($k=2$, positive sequence – coincide to a_1), and

a_{1m} - represents the positive sequence component of the current for $k=q$, (coinciding to a_1).

Similarly, for (a_{21} , a_{22} , ..., a_{2m}) will coincide to controllable input 1 (a_2) alternative, (a_{31} , a_{32} , ..., a_{3m}) will coincide to a_3 , etc.

This will become clearer, once the decision table has been developed.

Of next importance, is determining the capacitor's size % Q_C values required to tune the end-user network to resonate at a characteristic harmonic frequency [fr(hch)] within the "RFB" and to compare them to the % Q_C step sizes used to establish the resonant points within the outer limits of "RFB", namely % Q_C [fr(hch)] values.

Therefore: let % Q_C [fr(hch)] represent the capacitor size to tune the network to resonate at a fr(hch) within the RFB, namely:

$$\% Q_C [\text{fr(hch)}] = \text{capacitor size} \quad (4.16)$$

$$\text{subject to:} \quad 0\% Q_C < \% Q_C \leq 100\% Q_C \quad (4.17)$$

Next, we use scan studies and generate resonant curves for the % Q_C [fr(hch)] values. In a similar way to that used for the RFB we establish a range called the "Characteristic Harmonic Resonance Frequency Band (CHRFB)". The CHRFB are those curves with apexes within the RFB. This then establishes which of the characteristic harmonics could be of concern when installing a chosen capacitor size. For example, in figure 4.6, it can be seen that the 5th, 7th, 11th, 13th, 17th and 19th characteristic harmonics fall within the RFB and could be of concern. If the drive is 6-pulse, then for figure 4.6:

$$a_{nm} = \text{hch} = 6k \pm 1 \quad k = 1, 2 \text{ and } 3 \text{ only} \quad (4.18)$$

This then is a way of narrowing the range of harmonics that need to be considered for the decision model. Thus,

DECISION ALTERNATIVE	
CONTROLLABLE INPUT 2	(hch)
a_{11}	5 th
a_{12}	7 th
a_{13}	11 th
a_{14}	13 th
a_{15}	17 th
a_{16}	19 th

Table 4.1 Controllable input 2, alternatives per % Q_C size

CHAPTER 4

By using the “RFB” and “CHRFB” together it is possible to constrain the number of characteristic harmonics to be applied to the decision model as:

$$\text{CHRFB} < \text{RFB} \quad (4.19)$$

STEP 3:

We now need to see if we can constrain the capacitor size alternatives (a_n) within range 1 (equation 4.6).

Power engineers supply electric power via transformers and distributors to end-users. Each must be able to handle the current required. The more current, the larger and more expensive is the equipment and the greater the I^2R losses. So, it is advantageous to the utility to have its end-users run with improved dpf's. Many utilities encourage end-users to operate at a minimum dpf.

The recommended rule of thumb for a minimum dpf is 90% [32].

Let % $Q_{C(\text{MIN})}$ represent a dpf of 90%, then the range of capacitor size alternatives can be constrained to:

$$\text{Range 2} = \% Q_{C(\text{MIN})} \leq \%Q_C \leq \% Q_{C(\text{MAX})} \quad (4.20)$$

Obviously, within range 2, there are an infinite number of decision alternatives for % Q_C .

As stated before, the number can be limited to a finite number, but there must be at least two numbers within a set, otherwise there is no decision problem. How this is applied will be demonstrated in the case studies conducted in the following chapters and generally depends on the RFB and CHRFB relevant to a given investigation.

STEP 4:

We have now considered the inputs, which are under the control of the decision-maker.

Therefore, we can say that when considering the severity of harmonic resonance for a decision model, there are 2 – controllable input as shown in table 4.2:

DECISION ALTERNATIVES	
CONTROLLABLE INPUTS	
1	2
a_1 $= \%Q_{C(a1)}$	a_{11} a_{12} . . . a_{1m}
a_2 $= \%Q_{C(a2)}$. . .	a_{21} a_{22} . . . a_{2m}
. . . a_n $= \%Q_{C(a_n)}$	a_{n1} a_{n2} . . . a_{nm}

Table 4.2 Decision alternatives 2-controllable inputs

4.7 OBJECTIVE FUNCTION (BLOCK D)

The objective is to determine the severity of resonance at key harmonic frequencies. For decision analysis purposes the objective must be expressed mathematically (quantitatively) and be capable of also being expressed qualitatively. [Chapter 3, section 3.6 (c)].

There is thus a need for a mathematical expression, which quantifies harmonic resonance and gives an indication of the severity of resonance that a capacitor is subjected to in an end-user plant.

The term severity of harmonic resonance is not defined in literature. It's meaning must therefore be formulated before it can be expressed mathematically.

Capacitors are sensitive to over-voltages and this can be due to harmonic resonance.

Let, $V_{C(hch)}$ represent the rms voltage across the capacitor in an end-user plant when characteristic harmonics are injected into the system. Therefore, $V_{C(hch)}$ across the capacitor is used as the basis for defining the performance measure (outcome) for the decision model.

If resonance occurs at a characteristic harmonic frequency [$fr = hch$], the voltage $V_{C(hch)}$ at the resonant point will be a maximum. At the same time there will also be voltages $V_{C(hch)}$ at the other characteristic harmonic frequencies [$fr \neq hch$]. If, the impedance parameters cause resonance at a frequency other than at a characteristic harmonic

[fr≠hch], the voltages $V_{C(hch)}$ at the injected frequencies will be less than the value at resonance.

The initial step when formulating the outcome for a decision model is to define in broad terms an explicit performance measure that gauges the attainment of the objective.

As the severity of harmonic resonance is the objective, let $HRSI_{(hch)}$ represent this quantity and let it be stated in words as follows [6]:

$$HRSI_{(hch)} = \frac{\text{Voltages across the capacitor in the end-user plant when not resonating at Characteristic harmonic frequency.}}{\text{Voltages across the capacitor in the end-user plant when resonating at a characteristic harmonic frequency}} \quad (4.21)$$

The “Harmonic Resonance Severity Index [$HRSI_{(hch)}$], is defined as the ratio of the capacitor voltage [$V_{C(hch)}$] when the end-user plant is not resonating at a characteristic harmonic frequency [fr≠hch] to the capacitor voltage $V_{C(hch)}$ when it is resonating at a characteristic harmonic frequency [fr = hch].

The outcome (HRSI) for the decision model must be mathematically related to the 2-controllable inputs (a_n) and (a_{nm}) and states of nature (s_N) by means of an equation. This equation is called the objective function for the decision problem and takes into account constraints that lead to a meaningful solution.

Using equation 4.21 and taking the relevant variables into account, the objective function is [6]:

$$\%HRSI_{(sN)(anm)} = \frac{V_{C(sN)(anm)}[\%Q_{C(an)}(fr \neq hch)]}{V_{C(sN)(anm)}[\%Q_{C(an)}(fr = hch)]} \times 100\% \quad (4.22)$$

Note:

- If the capacitor size $\%Q_C$ is such that resonance (fr) does not occur at a characteristic harmonic, then the severity of harmonic resonance will be less than 100% ($\%HRSI < 100\%$).
- If, the capacitor size $\%Q_C$ causes resonance (fr) at a characteristic harmonic, then the severity of harmonic resonance will be 100 % ($\%HRSI = 100\%$) at that characteristic harmonic frequency (e.g., hch = 5th harmonic). However, the severities at the other characteristic harmonics (e.g., 7th, 11th, etc) will be less than 100% ($\%HRSI < 100\%$).

CHAPTER 4

- c. If, the numerator value is bigger than the denominator value in equation (4.22), then the severity of harmonic resonance will be greater than 100% (%HRSI>100%). For example, this could happen when the plant is operating under minimum demand conditions.

For decision analysis purposes, the following qualitative categories are allocated to the %HRSI values [6]:

%HRSI CATEGORIES	
%HRSI	QUALITATIVE CATEGORY
$\geq 100\%$	Very severe
75% - 99%	Severe
50% - 74%	Not severe
0% - 49%	Least severe

Table 4.3 %HRSI Qualitative categories

4.8 DECISION TABLE TO STRUCTURE AND REPRESENT THE HARMONIC RESONANCE DECISION PROBLEM

After having formulated the objective function and defined the decision alternatives, states of nature and the outcome, a decision table can be developed to structure and represent the harmonic resonance decision problem, namely [6]:

SEVERITY OF HARMONIC RESONANCE-DECISION TABLE					
DECISION ALTERNATIVES		STATES OF NATURE			
CONTROLLABLE INPUTS		UNCONTROLLABLE INPUTS			
1	2	s_1	s_2	$s_3 \dots\dots$	$\dots\dots s_N$
a_1 $= \%Q_{C(a_1)}$	a_{11}	$HRSI_{(s_1)(a_{11})}$	$HRSI_{(s_2)(a_{11})}$	$HRSI_{(s_3)(a_{11})}$	$HRSI_{(s_N)(a_{11})}$
	a_{12}	$HRSI_{(s_1)(a_{12})}$	$HRSI_{(s_2)(a_{12})}$	$HRSI_{(s_3)(a_{12})}$	$HRSI_{(s_N)(a_{12})}$

	a_{1m}	$HRSI_{(s_1)(a_{1m})}$	$HRSI_{(s_2)(a_{1m})}$	$HRSI_{(s_3)(a_{1m})}$	$HRSI_{(s_N)(a_{1m})}$
a_2 $= \%Q_{C(a_2)}$	a_{21}	$HRSI_{(s_1)(a_{21})}$	$HRSI_{(s_2)(a_{21})}$	$HRSI_{(s_3)(a_{21})}$	$HRSI_{(s_N)(a_{21})}$
	a_{22}	$HRSI_{(s_1)(a_{22})}$	$HRSI_{(s_2)(a_{22})}$	$HRSI_{(s_3)(a_{22})}$	$HRSI_{(s_N)(a_{22})}$

	a_{2m}	$HRSI_{(s_1)(a_{2m})}$	$HRSI_{(s_2)(a_{2m})}$	$HRSI_{(s_3)(a_{2m})}$	$HRSI_{(s_N)(a_{2m})}$
a_n $= \%Q_{C(a_n)}$	a_{n1}	$HRSI_{(s_1)(a_{n1})}$	$HRSI_{(s_2)(a_{n1})}$	$HRSI_{(s_3)(a_{n1})}$	$HRSI_{(s_N)(a_{n1})}$
	a_{n2}	$HRSI_{(s_1)(a_{n2})}$	$HRSI_{(s_2)(a_{n2})}$	$HRSI_{(s_3)(a_{n2})}$	$HRSI_{(s_N)(a_{n2})}$

	a_{nm}	$HRSI_{(s_1)(a_{nm})}$	$HRSI_{(s_2)(a_{nm})}$	$HRSI_{(s_3)(a_{nm})}$	$HRSI_{(s_N)(a_{nm})}$
PROBABILITIES		$p(s_1)$	$p(s_2)$	$p(s_3)$	$p(s_N)$

Table 4.4 Severity of harmonic resonance decision table

4.9 APPLICATION OF UTILITY THEORY (BLOCK E)

Stage 2 of the 3-stage process is now applied. As probabilities are assigned to the states of nature, decision-making under risk can be made (deterministic model). As the outcomes (%HRSI) are percentages and are non-monetary values, utility theory needs to be applied to select the decision alternative that meets the objective.

Each outcome in the decision table is replaced by their utility value. The table then becomes a “utility table” and has the following general format:

UTILITY TABLE FOR MAKING A DECISION UNDER RISK					
DECISION ALTERNATIVES		STATES OF NATURE			
CONTROLLABLE INPUTS		UNCONTROLLABLE INPUTS			
1	2	S_1	S_2	$S_3 \dots\dots$	$\dots\dots S_N$
a_1 $= \%Q_C(a_1)$	a_{11}	$U_{(s1)(a11)}$	$U_{(s2)(a11)}$	$U_{(s3)(a11)}$	$U_{(sN)(a11)}$
	a_{12}	$U_{(s1)(a12)}$	$U_{(s2)(a12)}$	$U_{(s3)(a12)}$	$U_{(sN)(a12)}$

	a_{1m}	$U_{(s1)(a1m)}$	$U_{(s2)(a1m)}$	$U_{(s3)(a1m)}$	$U_{(sN)(a1m)}$
a_2 $= \%Q_C(a_2)$	a_{21}	$U_{(s1)(a21)}$	$U_{(s2)(a21)}$	$U_{(s3)(a21)}$	$U_{(sN)(a21)}$
	a_{22}	$U_{(s1)(a22)}$	$U_{(s2)(a22)}$	$U_{(s3)(a22)}$	$U_{(sN)(a22)}$

	a_{2m}	$U_{(s1)(a2m)}$	$U_{(s2)(a2m)}$	$U_{(s3)(a2m)}$	$U_{(sN)(a2m)}$
. . . a_n $= \%Q_C(a_n)$	a_{n1}	$U_{(s1)(an1)}$	$U_{(s2)(an1)}$	$U_{(s3)(an1)}$	$U_{(sN)(an1)}$
	a_{n2}	$U_{(s1)(an2)}$	$U_{(s2)(an2)}$	$U_{(s3)(an2)}$	$U_{(sN)(an2)}$

	a_{nm}	$U_{(s1)(anm)}$	$U_{(s2)(anm)}$	$U_{(s3)(anm)}$	$U_{(sN)(anm)}$
PROBABILITIES		$P_{(s1)}$	$P_{(s2)}$	$P_{(s3)}$	$P_{(sN)}$

Table 4.5 Utility Table for making a decision under risk

Utility theory is about preferences and desirability (chapter 3, section 3.18). The following “utility theory method” is introduced to assist with the decision-making [6]:

- Derive a “relevant decision table” (e.g., table 3.7). Then identify and rank the best and worst %HRSI values [best = lowest %HRSI, worst = highest %HRSI]. The highest %HRSI outcome value is not the most desirable.
- Assign a utility value, “ $U(\%HRSI)$ ” of “1” and “0” to the best and worst severities, respectively.

CHAPTER 4

- c. Derive a utility function $[U(\%HRSI)]$ versus $\%HRSI$ for the decision maker. A concave shape (risk averse) utility function can be expected, as decision makers on resonance would normally avoid risky options. As an “innovation”, plot a risk neutral utility function first (see figure 3.15) and use this as a guideline to assist with deriving a concave curve. A risk averse descending utility function is expected as the lowest $\%HRSI$ is the best outcome and the highest $\%HRSI$ is the worst outcome. Any severity outcomes can be used, provided that they fall within the best and worst outcomes for the decision problem investigated or the outcomes from the decision table can be used. Utilities are determined as follows:
- (i) Apply the “variable probability method” to elicit probability indifference “ p_{ind} ” values for the decision maker, namely: (chapter 3, section 3.18.5)

ELICITATION SESSION
<p>In general: Offer the decision maker the choice:</p> <p>A: $\%HRSI_{(SN)(anm)}$ for certain, or</p> <p>B: entering a hypothetical game which results in either: (1) a “p” chance of obtaining the best $\%HRSI_{(SN)(anm)}$ outcome, or (2) a “$1-p$” chance of obtaining the worst $\%HRSI_{(SN)(anm)}$ outcome.</p> <p>A series of these questions are asked, until the decision maker is indifferent (equally attracted) between the certain severity value and the game value[“p_{ind}”].</p>

Table 4.6 Elicitation session

- (ii) Calculate the utility values: (chapter 3, equation 3.19)

$$U(\%HRSI)(\text{sure outcome}) = p_{(ind)} U(\%HRSI)(\text{best outcome}) + (1 - p_{(ind)}) U(\%HRSI)(\text{worst outcome}) \quad (4.23)$$

A $U(\%HRSI)$ is equal to the point of indifference provided “ p ” is between 1 and 0.

- (iii) Plot the utility function. Read off the utility values and represent them on a “relevant utility table”.
- d. Calculate the expected utility (EU) value for each decision, (chapter 3, equation 3.13):

$$EU_{(an)(anm)} = \sum_{N=1} p_{(sN)} U(\%HRSI)_{(sN)(anm)} \quad (4.24)$$

- e. Select the highest $EU_{(an)(anm)}$ value and this gives the size of capacitor $\%Q_{C(an)(anm)}$ to be installed.
- f. For this size, determine from the decision table the severities of resonance for the states of nature(s_N). Then make and record “**decision 1**”.

Decision 1

The derivations of the %HRSI outcomes in the decision table serve as the initial phase in the process of making a mitigation decision (decision 1). After applying utility theory, the highest expected utility value (EU) is determined and this identifies the capacitor size (controllable input 1) to be installed $\%Q_{C(an)}$ (decision 1). The offending characteristic harmonic (a_{nm}) (controllable input 2) is also identified, namely:

DECISION 1 IN THE PROCESS OF MAKING A MITIGATION DECISION					
Decision alternatives		States of nature			Expected utility Value
Controllable inputs		Uncontrollable inputs			
1	2	S ₁	S ₂	...S _N	EU
a _n = %Q _{C(an)} (selected capacitor size)	a _{nm} (m th harmonic)	%HRSI _{(s1)(anm)}	%HRSI _{(s2)(anm)}	%HRSI _{(sN)(anm)}	EU _{(an)(anm)} (highest value)

Table 4.7 Mitigation decision 1

The %HRSI outcomes, by themselves, are inadequate for making the mitigation decision as the severity of resonance, although high, may not necessary cause damage to the chosen pf correction capacitor. A further stage is needed to assess if the severities will cause damage or not to the chosen capacitor (decision 2).

4.10 MAKING A MITIGATION DECISION ON HARMONIC RESONANCE (BLOCK F)

Harmonics can affect capacitor banks. Since their reactance decreases with frequency, capacitors act as sinks for harmonics. Resonance can cause harmonics to be amplified and the resultant voltages to exceed the rating of the capacitor, leading to capacitor damage or blown fuses. For these reasons, harmonic penetration results (chapter 2, section 2.4.7) rather than “scan study results” are used to evaluate capacitors in terms of international standards.

Both the IEEE and IEC bodies provide standard loading indices for pf capacitors. For convenience and ease of reference, these standards are summarized in Appendix 1 (Tables A1.1, A1.2 and A1.3, are important). Reference [3] suggests four IEEE standard loading indices and limits for capacitors. The IEC standard provides one limit and states that pf capacitors shall be suitable for continuous operation at an rms current of 1.30 times the current at rated sinusoidal voltage and frequency. This takes care of the combined effects due to harmonics and over-voltages [36]. Neither the IEEE nor the IEC indices (Table A1.3) directly reflect the severity of harmonic resonance nor do they take into account the states of nature and any decision alternatives.

There is thus a need for a new index which can be used for making a “**mitigation decision (decision 2)**”, where such an index not only takes the severity of harmonic resonance into account but also mathematically relates the 2-controllable inputs (decision alternatives), uncontrollable inputs (states of nature) and the outcomes (severity of harmonic resonance) together. The IEEE standard uses more than one index to evaluate capacitor loadings. The IEC standard, applies only one index.

CHAPTER 4

For the purpose of making decision 2, a new index based on the IEC standard is introduced for making a mitigation decision on pf capacitors in end-user plants. The IEEE indices can be used as a check. Let this index be called the “Mitigation Index (%MI)” and is used to assess the capability (not become damaged) of the pf capacitor under steady-state conditions, namely [6]:

$$\%MI_{\%HSRI(sN)(anm)} = \frac{I_{C(RMS)}}{1.30I_1} \times 100\% \quad (4.25)$$

$$\text{subject to:} \quad V_{C1} \leq 1.10U_N \quad (4.26)$$

$$\text{where:} \quad I_{C(RMS)} = \sqrt{\sum_{h=hch} (I_1^2 + I_h^2)} \quad (4.27)$$

$I_{C(RMS)}$ = capacitor current

$$I_1 = \text{rated fundamental frequency current for the capacitor.} \quad (4.28)$$

$$U_N = \text{nominal voltage for the capacitor.} \quad (4.29)$$

$$V_{C1} = \text{fundamental frequency capacitor voltage after installation.} \quad (4.30)$$

The developed index thus takes into account the main ingredients (%HSRI, s_N , a_{nm}) of the decision model (outcomes, states of nature, decision alternatives).

Decision 2

Once the %MI have been calculated for the chosen capacitor size (decision 1), decision 2 can be made and recorded as follows:

DECISION 2 IN THE PROCESS OF MAKING A MITIGATION DECISION				
s_N	a_{nm}	%HSRI	%MI	% V_{C1}
s_1		%HSRI $_{(s1)(anm)}$	%MI $_{(s1)(anm)}$	% $V_{C1(s1)(anm)}$
.	m^{th} harmonic	.	.	.
.		.	.	.
.		.	.	.
s_N		%HSRI $_{(sN)(anm)}$	%MI $_{(sN)(anm)}$	% $V_{C1(sN)(anm)}$

Table 4.8 Mitigation decision 2

If % MI $\geq 100\%$, check that $V_{C1} \leq 1.10U_N$. If so, there is no need for mitigation. If $V_{C1} > 1.10U_N$, the IEC standard stipulates (Appendix 1) that the dominating harmonic must be identified in order to find the best remedy as there is a need for mitigation. This is where the usefulness of the decision theory process, %HSRI and %MI is seen.

Figure 4.7 shows a “template” for making a mitigation decision on harmonic resonance:

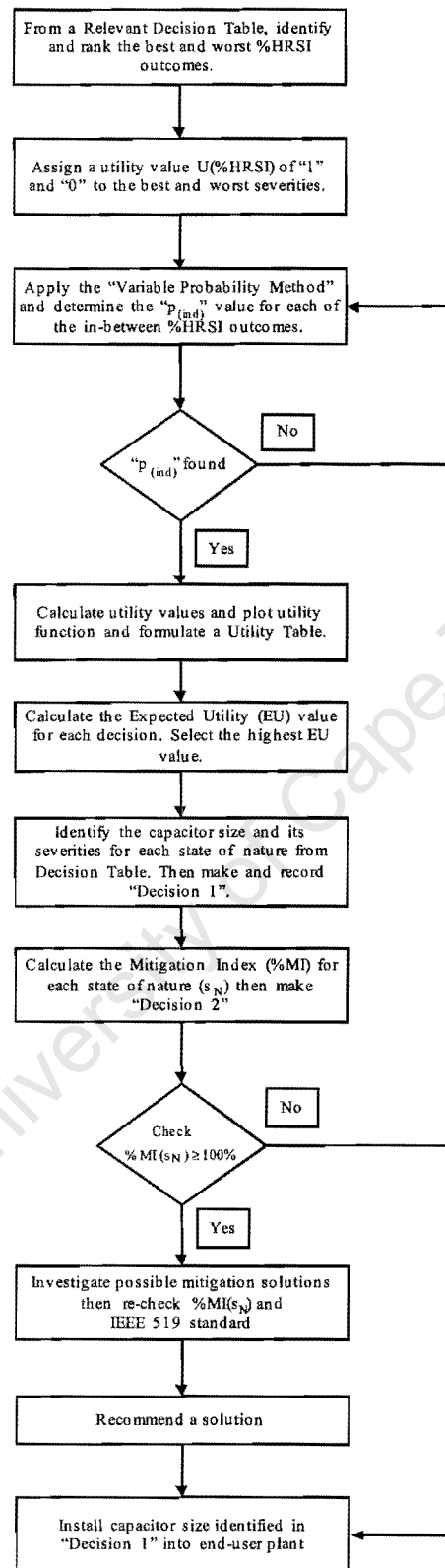


Figure 4.7 Template for making a mitigation decision on Harmonic Resonance

4.11 SUMMARY

A new decision theory process for making a mitigation decision on harmonic resonance has been developed. A 2-controllable input approach is introduced for identifying decision alternatives. The Harmonic Resonance Severity Index (%HRSI) is formulated as the objective function for the decision model. A new decision table is developed to structure and represent decision problems on harmonic resonance. A new application for utility theory is introduced for assisting in choosing the most desirable capacitor size (decision 1). A new index which takes into account decision alternatives, states of nature and outcomes (severity of harmonic resonance) of the decision-model, called the mitigation index is developed to help make the decision as to whether or not mitigation is needed (decision 2). A template for making a mitigation decision on harmonic resonance is developed. The theoretical background for making a mitigation decision, based on decision theory, has been developed and introduced as a foundation for case studies, which follow in the next chapters.

CHAPTER 5

CASE STUDY 1

APPLICATION OF DEVELOPED DECISION THEORY PROCESS TO A SIMPLE POWER SYSTEM

In this chapter, the theory developed in chapter 4 is applied to a case study to demonstrate the effectiveness/usefulness of the developed three-stage decision theory process for making a mitigation decision. A simple power system made up of a radial distribution network, supplying an end-user plant containing one harmonic source, is used. A capacitor bank is to be added to the system to improve the power factor. Each of the three stages of the developed decision theory process is applied in detail using a step-by-step approach with attention to detail. Calculations are shown. For completeness, Appendices 2 to 5 accompany this chapter. The computer model for conducting harmonic “scan” and “penetration” studies are given in Appendices 2 and 3, respectively. The calculations for the %HRSI outcomes are in Appendix 4. In Appendix 5, calculations needed for the %MI value are recorded. A utility function is derived for the decision-maker taking his preferences into account and a utility table is developed for making decision 1, namely to choose the capacitor size which best meets the objective. %MI values for the states of nature are calculated. The finding based on %MI results is checked against IEEE standards for capacitor loadings and recorded in Appendix 5. The chapter ends with decision 2, that is, whether mitigation is needed or not.

5.1 ONE-LINE DIAGRAM FOR CASE STUDY 1

Figure 5.1 shows the one-line-diagram for case 1.

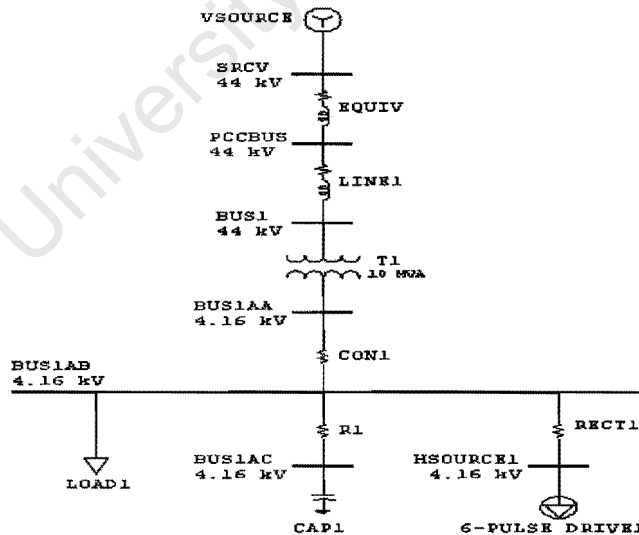


Figure 5.1 One-line-diagram – case 1

In this case study (test system), a radial power system is used. This is a system to be built in the future. A distribution network will supply an end-user plant having only one

harmonic source. The only capacitance in the system will be the pf correction capacitor bank installed in the plant. The theory and results relevant to this case was published in reference [7].

The end-user plant is to be supplied from a 44kV/4.16kV radial distribution network. The supply transformer is to be rated at 10MVA and have a X/R ratio of 5. The plant will be comprised of a linear load of 4.5 MVA, 0.8 dpf (Load 1) and a 6-pulse drive of 2.574MVA will be added. A capacitor bank (CAP 1) is to be added to the plant to improve the dpf.

5.2 DECISION SCENARIO

The following decision scenario requires investigation.

“The end-user wants to introduce a pf correction capacitor in his plant, to improve the 0.8 dpf. He realizes that the 6-pulse drive in his plant could cause resonance at a characteristic harmonic, which may require the installation of a mitigation device at his expense. A further concern is that the level of power demand drawn by the plant is uncertain at this stage. Guidelines from the future engineer is that the plant will operate at either a minimum, a mean or at a full load demand level”.

5.3 STAGE 1 OF CASE 1– QUANTITATIVE DECISION MODEL

Stage 1 is about building a decision making model and the first steps are to define the problem and identify the objectives.

5.3.1 DEFINE THE PROBLEM (BLOCK A)

The following question is asked: In an end-user plant which includes a harmonic producing source as shown in figure 5.1, will the resonance caused by the installation of a pf capacitor bank be severe enough to result in damage to the capacitor when the plant is operating at a minimum, mean or full load demand level?

5.3.2 IDENTIFY THE OBJECTIVES (BLOCK A)

- a. For the plant which will operate at either minimum, mean or full load demand conditions, the objective is to determine the severity of resonance at characteristic harmonic frequencies caused by the installation of pf capacitors. That is, make a decision (decision 1) between different sizes of capacitors as to which size should be chosen, taking into account the preferences/desires of the decision-maker.
- b. Further, if high levels of severity of harmonic resonance are found in the investigation and if severity falls within the very severe or severe categories of harmonic resonance, make a mitigation decision (decision 2).

5.3.3 IDENTIFY THE STATES OF NATURE (BLOCK B)

In our case study, the choice of the states of nature depends on future demand conditions. The possibilities are the mutually exclusive and collectively exhaustive events of minimum, mean or full load demand conditions. There are thus a finite number of three states of nature and each is expressed qualitatively.

The states of nature (s_N) under steady-state conditions are identified as follows: (qualitative identification).

$$\begin{aligned}s_1 &= \text{minimum load demand} \\ s_2 &= \text{mean load demand} \\ s_3 &= \text{full load demand}\end{aligned}$$

The qualitative states of nature need to be identified in quantitative terms as discussed in chapter 4, equation 4.3.

$$s_N = n \times s_3 \quad 0 \leq n \leq 1 \quad (5.1)$$

As the demand levels “n” are uncertain, and as the plant is still to be built, (no measurement data available) the future plant engineer is asked to use his past experience (chapter 3, figure 3.4) to estimate the n^{th} values for the identified states of nature. He estimates that $n = 0.25$ and 0.6 for s_1 and s_2 , respectively;

$$\begin{aligned}s_1 &= 0.25 \text{ full load demand} \\ s_2 &= 0.6 \text{ full load demand} \\ s_3 &= 1.0 \text{ full load demand}\end{aligned}$$

therefore:

$$S_{E(s1)} = P_{E(s1)} + j Q_{E(s1)} \quad (5.2)$$

$$S_{E(s2)} = P_{E(s2)} + j Q_{E(s2)} \quad (5.3)$$

$$S_{E(s3)} = P_{E(s3)} + j Q_{E(s3)} \quad (5.4)$$

The dpf's are $\cos\phi_{E(s1)}$, $\cos\phi_{E(s2)}$ and $\cos\phi_{E(s3)}$, respectively.

To make decisions in the risk zone and obtain a deterministic model it is essential to elicit and assign probabilities to the three states of nature.

Probabilities can be found objectively or subjectively. If the plant physically exists, probabilities (a posteriori) can be objectively obtained. If the plant is still to be built (a priori), the subjective assessment method of relative heights must be applied to elicit

CHAPTER 5

probabilities from discrete or continuous probabilities (chapter 3, section 3.14 and examples 3.2 and 3.4).

As the plant is still to be built and as discrete values have been identified for the n^{th} values of the states of nature, the engineer is questioned so that a discrete probability distribution can be derived by means of the method of relative heights.

The full load rating of the plant is 4.5 MVA, therefore:

$$s_1 = 1.125 \text{ MVA}$$

$$s_2 = 2.700 \text{ MVA}$$

$$s_3 = 4.500 \text{ MVA}$$

After questioning the engineer, $s_2 = 2.7 \text{ MVA}$ is identified as the most likely demand and 10 units on the graph represent this as shown in figure 5.2. He further reveals that the minimum demand (s_1) is half as likely as the mean demand (s_2) and is represented by 5 units on the graph. He also reveals that the full demand (s_3) is one tenth as likely as s_2 and is represented on the graph by 1 unit.

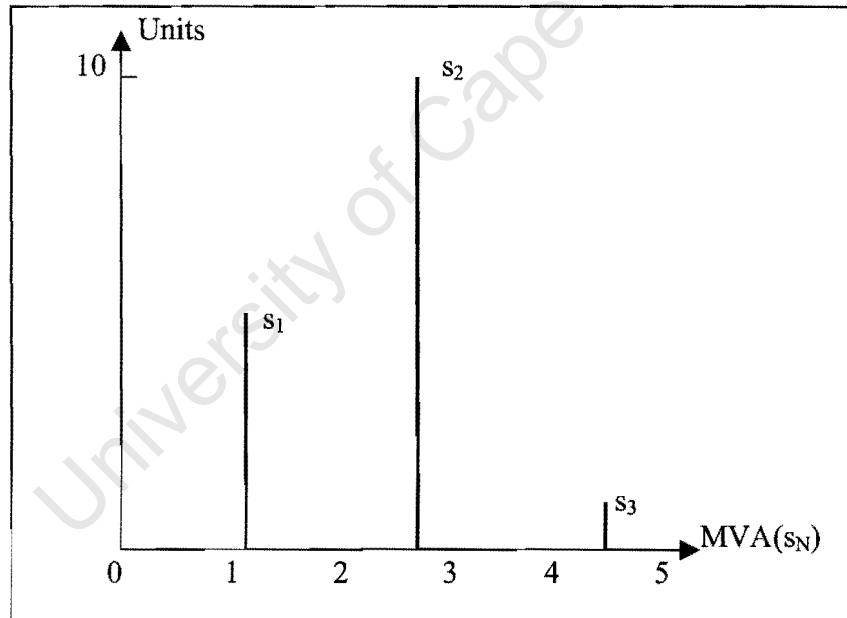


Figure 5.2 Frequency distribution (S_E)

Since, the vertical lines (L) represent probabilities, they must be within the range.

$$0 \leq p_{(sN)} \leq 1 \quad (5.5)$$

and,

$$\sum p_{(sN)} = 1 \quad (5.6)$$

then, normalizing, $(L/\sum L)$

$$\begin{aligned} p(1.125\text{MVA}) &= 5/16 = 0.3125 \\ p(2.700\text{MVA}) &= 10/16 = 0.6250 \\ p(4.500\text{MVA}) &= 1/16 = 0.0625 \end{aligned}$$

If rounded off to one decimal place, then the probabilities are:

$$\begin{aligned} p_{(s1)} &= p(1.125\text{MVA}) = 0.3 \text{ (30\%)} \\ p_{(s2)} &= p(2.700\text{MVA}) = 0.6 \text{ (60\%)} \\ p_{(s3)} &= p(4.500\text{MVA}) = 0.1 \text{ (10\%)} \end{aligned}$$

Therefore, the discrete probability distribution for the plant (S_E) is shown in figure 5.3:

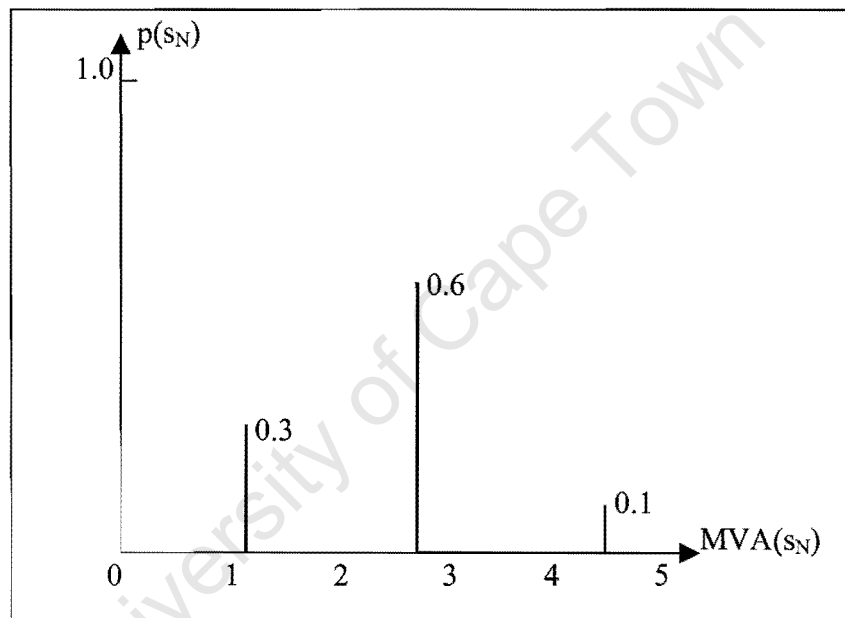


Figure 5.3 Discrete Probability Distribution (S_E)

5.3.4 IDENTIFY DECISION ALTERNATIVES (BLOCK C)

The following methodology is used to identify the decision alternatives:

STEP 1:

Develop a computer model for the system using the harmonic analysis software package chosen, with CAP 1 and drive 1 (harmonic source) installed. (see Appendix 2). The model given in Appendix 2 is for a $\%Q_C = 75\%$ and is a typical example of a model used for “Harmonic Scan” studies.

CHAPTER 5

By changing the value for %Q_C different correction levels are possible and various scans (resonance curves) can be generated.

For correction levels %Q_C [0%, 6.25%(168.75kvar), 12.5%(337.5kvar), 25% (675kvar), 50%(1350kvar), 75%(2025kvar) and 100%(2700kvar)], carry out a “Harmonic Scan” study.

Superimpose their resonance curves on a common set of axes [voltage (V) versus frequency in per-unit (Hpu)] and determine their resonance frequency (fr).

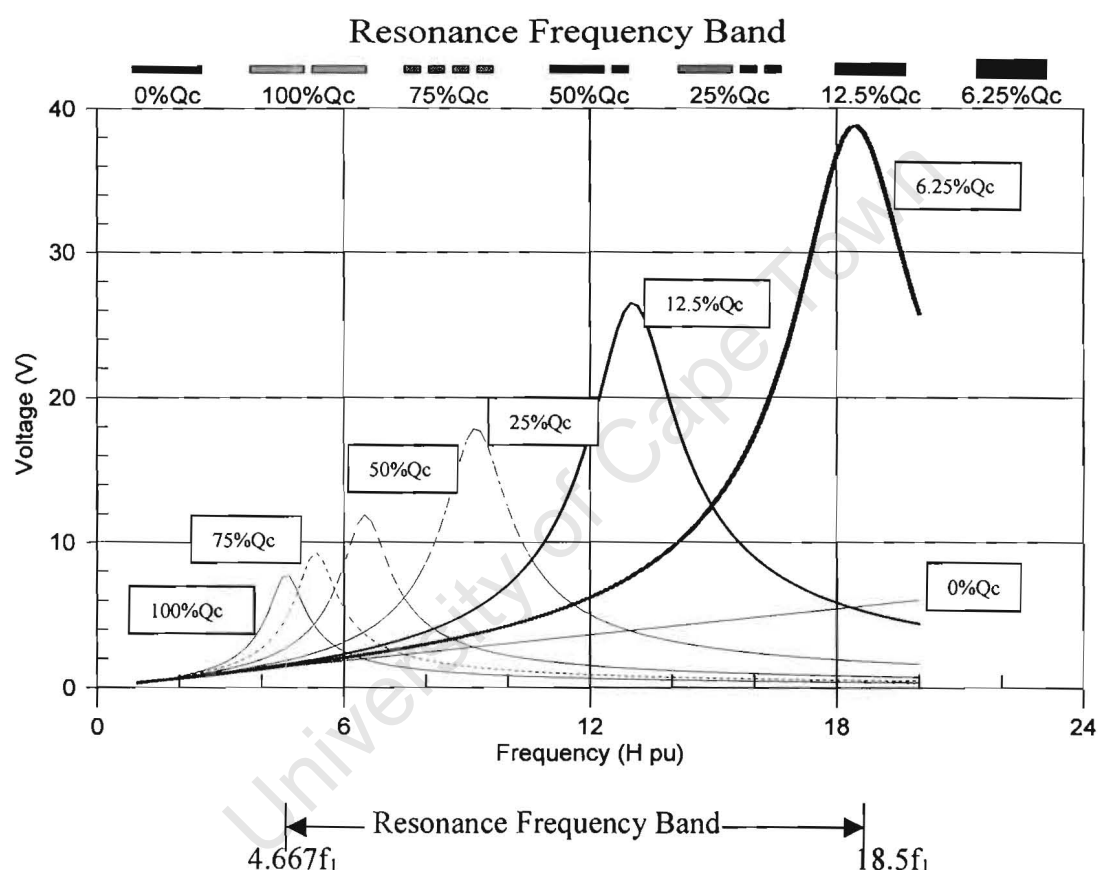


Figure 5.4 Resonance frequency band

Using the outer limits of fr(100%) and fr(6.25%), identify the “Resonant Frequency Band”(RFB), namely:

RFB \approx 100.00%Q_C(fr=4.667f₁), 75.00%Q_C(fr=5.330f₁), 50.00%Q_C(fr = 6.500f₁), 25.00%Q_C(fr = 9.167f₁), 12.50%Q_C(fr = 13.000f₁), 6.25%Q_C(fr = 18.500f₁).

The corresponding dpf band is: unity dpf, 0.982dpf, 0.936dpf, 0.871dpf, 0.836dpf and 0.818dpf.

CHAPTER 5

The apexes of the resonant points are found to fall on a straight line. (see chapter 4, equation 4.10). The following method is used to prove this statement.

$$V_D = \frac{dV_D}{dfr(h)} fr(h) + K \quad (5.7)$$

where: V_D is the driving point voltage at $fr(h)$ and “K” is a constant.

Determine the co-ordinates of the apexes of the resonant points using the cross-hair facility of the Scan Program (e.g., V_1 is the driving point voltage for 6.25%Qc and $fr(h) = 18.500f_1$):

Resonant Point Co-ordinates		
%Q _c	fr(h)	V _D
6.25	18.500	V ₁ = 38.813
12.50	13.000	V ₂ = 26.488
25.00	9.167	V ₃ = 17.833
50.00	6.500	V ₄ = 11.889
75.00	5.333	V ₅ = 9.299
100.00	4.667	V ₆ = 7.070

Table 5.1 Resonant point co-ordinates

To prove that the co-ordinates (last column in table 5.1) are correct, the following steps are used:

- (1) Determine the slope of the line.

$$V_2 = 26.488 \text{ V}, V_3 = 17.833 \quad \Delta V = V_2 - V_3 = 8.655 \text{ V}$$

$$\Delta fr(h) = 13 - 9.167 = 3.833$$

$$\text{Slope} = \frac{dV_D}{dfr(h)} = \frac{8.655}{3.833} = 2.258 \approx 2\frac{1}{4} \approx \frac{9}{4}$$

- (2) Determine the value for “K” from equation (5.7), using V_1 co-ordinates

$$K = 38.813 - [(2.258)(18.5)] = -2.960$$

- (3) Therefore the equation for the straight line is:

$$V_D = 2.258fr(h) - 2.960 \quad (5.8)$$

CHAPTER 5

- (4) Substitute $fr(h)$ values to check and prove that resonant point apexes within the RFB fall on a straight line, namely:

$$\begin{aligned} V_1 &= (2.258)(18.500) - 2.960 = 38.813V \text{ (plot = 38.813V)} \\ V_2 &= (2.258)(13.000) - 2.960 = 26.393V \text{ (plot = 26.488V)} \\ V_3 &= (2.258)(9.167) - 2.960 = 17.738V \text{ (plot = 17.833V)} \\ V_4 &= (2.258)(6.500) - 2.960 = 11.716V \text{ (plot = 11.889V)} \\ V_5 &= (2.258)(5.333) - 2.960 = 9.074V \text{ (plot = 9.299V)} \\ V_6 &= (2.258)(4.667) - 2.960 = 7.577V \text{ (plot = 7.070V)} \end{aligned}$$

Equation 5.8 can therefore be used for calculating intermediate resonant points within the RFB.

STEP 2:

Identify the characteristic harmonics (injected by the drive) that fall within the “RFB”.

The harmonics of concern are therefore the 5th, 7th, 11th, 13th and 17th. The 19th is just outside the RFB. As we prefer to work with harmonic pairs ($6k \pm 1$, $k=1,2$ and 3 only), the 19th harmonic is taken to be part of the RFB.

Calculate the $\%Q_C$ [$fr = hch$] value required to tune the system to resonate at a characteristic harmonic frequency [$fr(hch)$]. (Where hch are constrained to $k = 1,2$ and 3 only).

- (1) Calculate, using the equation for the straight line, the apex voltages for the resonant frequencies at the characteristic harmonics [$fr=(hch)$], that is:

$$\begin{aligned} V_{5th} &= (2.258)(5) - 2.960 = 8.3296V \\ V_{7th} &= (2.258)(7) - 2.960 = 12.8457V \\ V_{11th} &= (2.258)(11) - 2.960 = 21.8778V \\ V_{13th} &= (2.258)(13) - 2.960 = 26.3938V \\ V_{17th} &= (2.258)(17) - 2.960 = 35.4259V \\ V_{19th} &= (2.258)(19) - 2.960 = 39.9420V \end{aligned}$$

- (2). Calculate the driving point impedance at each $fr(hch)$. The scan uses a constant injection current of $1\angle 0^\circ A$, thus:

$$V_D = I_{(inject)} Z_{D(h)} \quad (5.9)$$

and

$$Z_{D(h)} = V_D \quad (5.10)$$

Thus, the $Z_{D(h)}$ will be the values calculated above.

- (3). Model the network without the capacitor (see Appendix 3).

CHAPTER 5

Then conduct a “Harmonic Penetration Study” to calculate the voltage at Bus1AB and the current I_h being injected by the drive.

Then determine the equivalent impedance $Z_h = V_{bus1AB} / I_h$ for the network:

IMPEDANCE VALUES AT LOAD BUS (BUS 1AB) ohms				
hch	V_{bus1AB} (V)	I_h (A)	Z_h (ohms)	$R_h + j X_h$ (ohms)
5	90.0558 \angle 162.577°	59.3620 \angle 82.6409°	1.51618 \angle 79.936°	0.2649 + j 1.4928
7	82.0398 \angle 123.706°	38.7690 \angle 43.0629°	2.11606 \angle 80.643°	0.3440 + j 2.0879
11	61.9749 \angle 42.6278°	18.6710 \angle -38.6827°	3.31927 \angle 81.310°	0.5014 + j 3.28117
13	51.2023 \angle -0.3574°	13.0560 \angle -81.8504°	3.92167 \angle 81.492°	0.58013 + j 3.8785
17	32.2920 \angle -95.4457°	6.29860 \angle -177.176°	5.12682 \angle 81.730°	0.7374 + j 5.0735
19	25.9200 \angle -149.602°	4.52380 \angle -231.314°	5.72962 \angle 81.712°	0.8258 + j 5.6697

Table 5.2 Impedance values

- (4). Draw the equivalent circuit for the network and plant including the pf correction capacitor bank, and calculate % Q_C [fr=(hch)] and $\cos\phi_E$ [fr=(hch)], as follows:

$$\frac{1}{X_{c(h)}} = \frac{1}{Z_{D(h)}} - \frac{1}{Z_h} \quad (5.11)$$

where: $X_{c(h)}$ is the reactance of the capacitor.

For the 5th Harmonic (hch = 5)

$$Y_{C(h)} = \frac{1}{8.3296 \angle 0^\circ} - \frac{1}{1.5168 \angle 79.936^\circ}$$

$$Y_{C(h)} = (0.12 + j 0) - (0.12 - j 0.649138) = 0.649138 \angle 90^\circ \text{ S}$$

$$X_{c(h)} = 1.540504 \angle -90^\circ \Omega \text{ at } 300\text{Hz}$$

$$X_{c(\omega_1)} = 5 \times 1.540504 = 7.70252 \angle -90^\circ \text{ at } 60\text{Hz}$$

$$I_{(\omega_1)} = (4160/\sqrt{3}) \div 7.70252 = 311.8169\text{A}$$

CHAPTER 5

$$Q_C[\text{fr}(5)] = [\sqrt{3}(4160)(311.8169)] \div 10^6 = 2.246 \text{ Mvars}$$

$$\%Q_C[\text{fr}(5)] = (2.246/2.7) \times 100\% = 83.2\% \text{ correction}$$

$$Q_E = 2700 - 2246 = 454 \text{ kvars} = 0.454 \text{ Mvars}$$

$$\phi_E = \tan^{-1} (454/3600) = 7.187^\circ$$

$$\cos \phi_E [\text{fr}(5)] = \cos 7.187^\circ = 0.992 \text{ dpf}$$

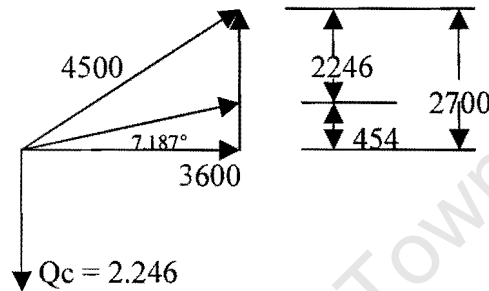


Figure 5.5 Power Triangle

Follow the same procedure and calculate the $\%Q_C [\text{fr}(\text{hch})]$ and $\cos \phi_E [\text{fr}(\text{hch})]$ values for the other characteristic harmonics. The following table gives the calculations:

CALCULATION OF $\%Q_C [\text{fr}(\text{hch})]$ AND $\cos \phi_E [\text{fr}(\text{hch})]$ VALUES						
Parameter	5	7	11	13	17	19
$Y_{C(h)} (\text{S})$	$0.6491 \angle -90^\circ$	$0.4662 \angle -90^\circ$	$0.2978 \angle -90^\circ$	$0.2521 \angle -90^\circ$	$0.1930 \angle -90^\circ$	$0.1727 \angle -90^\circ$
$X_{C(h)} (\Omega)$	$1.5405 \angle -90^\circ$	$2.1445 \angle -90^\circ$	$3.3578 \angle -90^\circ$	$3.9652 \angle -90^\circ$	$5.1806 \angle -90^\circ$	$5.7900 \angle -90^\circ$
$X_{C(\omega)} (\Omega)$	$7.7025 \angle -90^\circ$	$15.012 \angle -90^\circ$	$36.936 \angle -90^\circ$	$51.548 \angle -90^\circ$	$88.0710 \angle -90^\circ$	$110.010 \angle -90^\circ$
$I_{(\omega)} (\text{A})$	311.810	159.98	65.025	46.5922	27.2706	21.8320
$Q_C[\text{fr}(\text{hch})]$	2.246	1.1527	0.468	0.33571	0.1964	0.15730
$\%Q_C[\text{fr}(\text{hch})]$	83.200	42.6	17.35	12.4330	7.2700	5.8260
$Q_E (\text{Mvars})$	0.454	1.5473	2023148	2.3642	2.5035	2.5426
ϕ_E	7.187°	23.258	31.792°	33.2940°	34.8150°	35.2330°
$\cos \phi_E [\text{fr}(\text{hch})]$	0.992 dpf	0.918 dpf	0.849 dpf	0.8350 dpf	0.8209 dpf	0.8160 dpf

Table 5.3 $\%Q_C$ Calculations

“Harmonic Scan Studies” was used to visualize and prove that the calculations for $\%Q_C[\text{fr}(\text{hch})]$ give the correct tuning value $\text{fr}(\text{hch})$, namely as shown in figure 5.6:

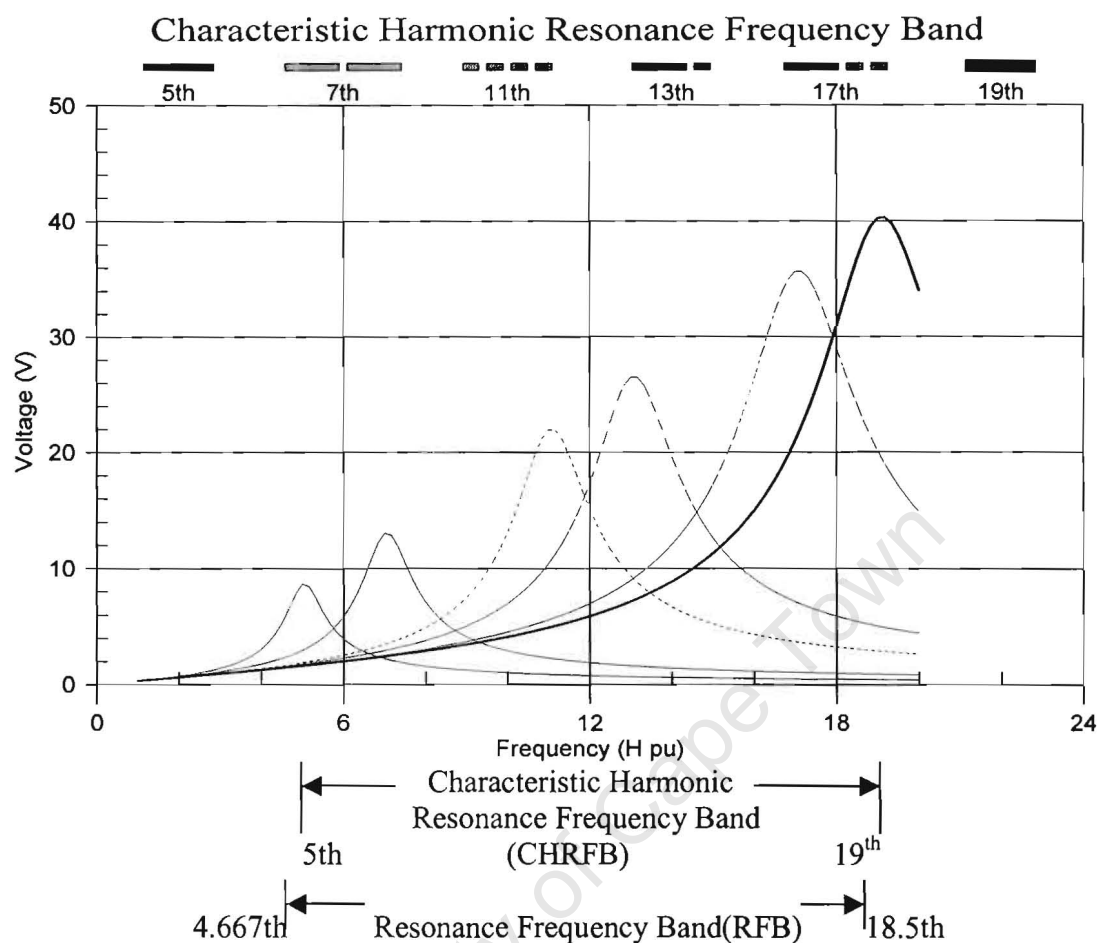


Figure 5.6 Characteristic Harmonic Resonance Frequency Band

Using the outer limits of $\%Q_{C(5th)}$ and $\%Q_{C(19th)}$, identify the “Characteristic Harmonic Resonant Frequency Band”(CHRFB), namely:

$$\text{CHRFB} \approx \%Q_{C(5th)} = 83.200\%, \%Q_{C(7th)} = 42.600\%, \%Q_{C(11th)} = 17.350\%, \%Q_{C(13th)} = 12.433\%, \%Q_{C(17th)} = 7.270\%, \%Q_{C(19th)} = 5.826\%.$$

It can be seen that the CHRFB falls within the RFB (including the 19th harmonic).

The purpose of the “CHRFB” is to constrain the number of characteristic harmonics, that is, the number of controllable input 2 alternatives for the decision model.

STEP 3:

Identify, when harmonics are present, the range within which a $\%Q_{C(an)}$ can be chosen, namely:

$$\text{Range 1} = 0\%Q_{C(\text{an})} < \%Q_{C(\text{an})} < 100\%Q_{C(\text{an})} \quad (5.12)$$

It is advantageous to operate with an improved power factor (chapter 4, equation 4.20). Therefore, the most likely range within which a $\%Q_C$ value would be chosen is:

$$\text{Range 2} = \%Q_{C(\text{an})(\text{MIN})} \leq \%Q_{C(\text{an})} \leq \%Q_{C(\text{an})(\text{MAX})} \quad (5.13)$$

where: $\%Q_{C(\text{an})(\text{MIN})}$ is the capacitor size to give a dpf of 0.9.

STEP 4:

Calculate the $\%Q_{C(\text{MIN})}$ value. The dpf for $\%Q_{C(\text{MIN})} = 0.9$, therefore $\phi_E = 25.84^\circ$. Using the power triangle (figure 5.5) but with the angle of 25.84° , $Q_E = 1743.5595$ kvars, therefore $Q_C = 2700 - 1743.5595 = 956.4404$ kvars and $\%Q_{C(\text{MIN})} = 35.4237\%$ correction.

Set the pf correction capacitor in the end-user computer model to the $\%Q_{C(\text{MIN})}$ value of 35.4237% (0.95644 Mvars). Carry-out a “Harmonic Scan Study” with Q_C set to $\%Q_{C(\text{MIN})}$ value and derive its resonant curve. Superimpose the scan results, but limit the plots to within Range 2, to visualize how the driving point voltages (V_D) and resonant curves change with variation in $\%Q_C$ value over Range 2 (figure 5.7):

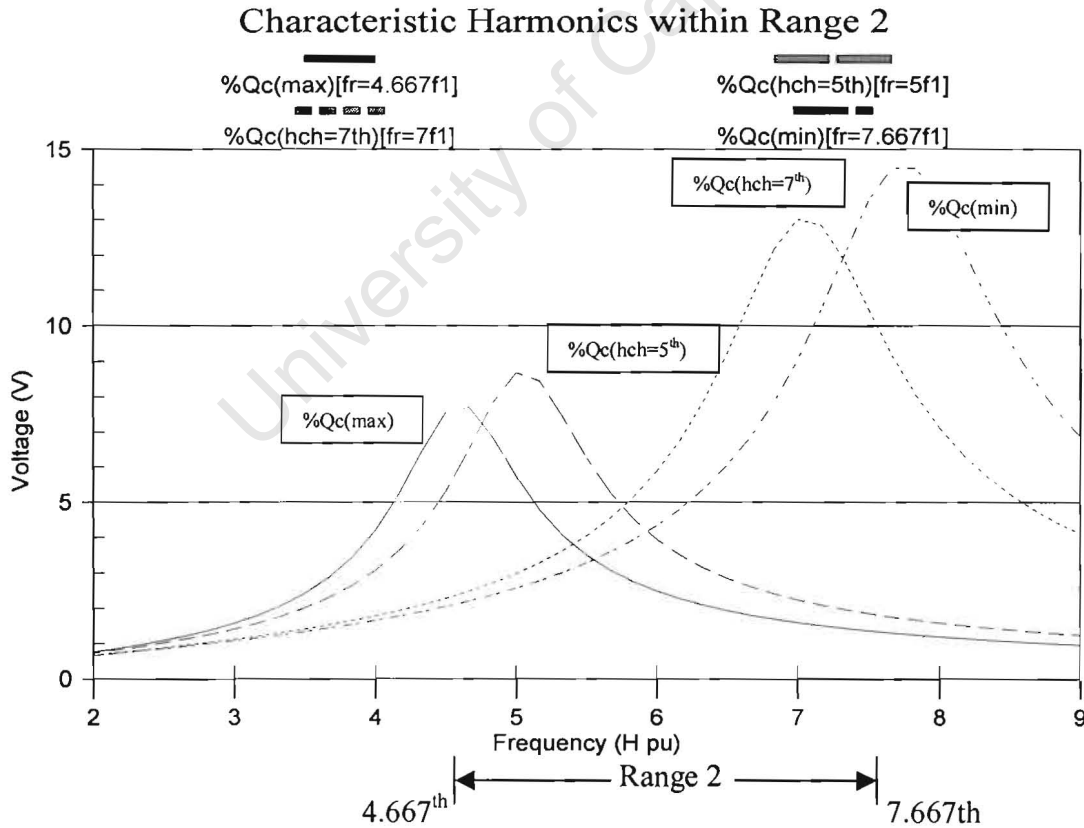


Figure 5.7 Characteristic Harmonics within Range 2

It can be seen that only the $\%Q_C(hch=5^{th})$ and $\%Q_C(hch=7^{th})$ characteristic harmonics capacitor sizes fall within range 2.

Obviously, there are an infinite number of decision alternatives (a_1, a_2, \dots, a_n) for $\%Q_C$ within range 2.

Note:

As stated in the introduction to identifying decision alternatives, these variables/inputs are courses of action to solve a problem. These variables/inputs are factors that influence the models outcome and are controllable inputs. In many cases, the problem may involve an unlimited number of decision alternatives. The nature of the problem usually will involve constraining the number of alternatives so that the inputs are meaningful, that is feasible. This means these variables/inputs (decision alternatives) can be limited to a finite number of decision alternatives. There must, however, be at least two decision alternatives within a set (a_1, a_2, \dots, a_n) representing a controllable input.

STEP 5:

Constrain the number of alternatives so that the inputs are meaningful and feasible.

Divide range 2 into three sub-regions, namely:

$$\text{Sub-region A} = \%Q_{C(an)(MIN)} \leq \%Q_{C(an)} < \%Q_{C(an)(7th)} \quad (5.14)$$

$$\text{Sub-region B} = \%Q_{C(an)(7th)} \leq \%Q_{C(an)} < \%Q_{C(an)(5th)} \quad (5.15)$$

$$\text{Sub-region C} = \%Q_{C(an)(5th)} \leq \%Q_{C(an)} \leq \%Q_{C(an)(MAX)} \quad (5.16)$$

Identify sub-regions A, B and C on the plot diagram (figure 5.8):

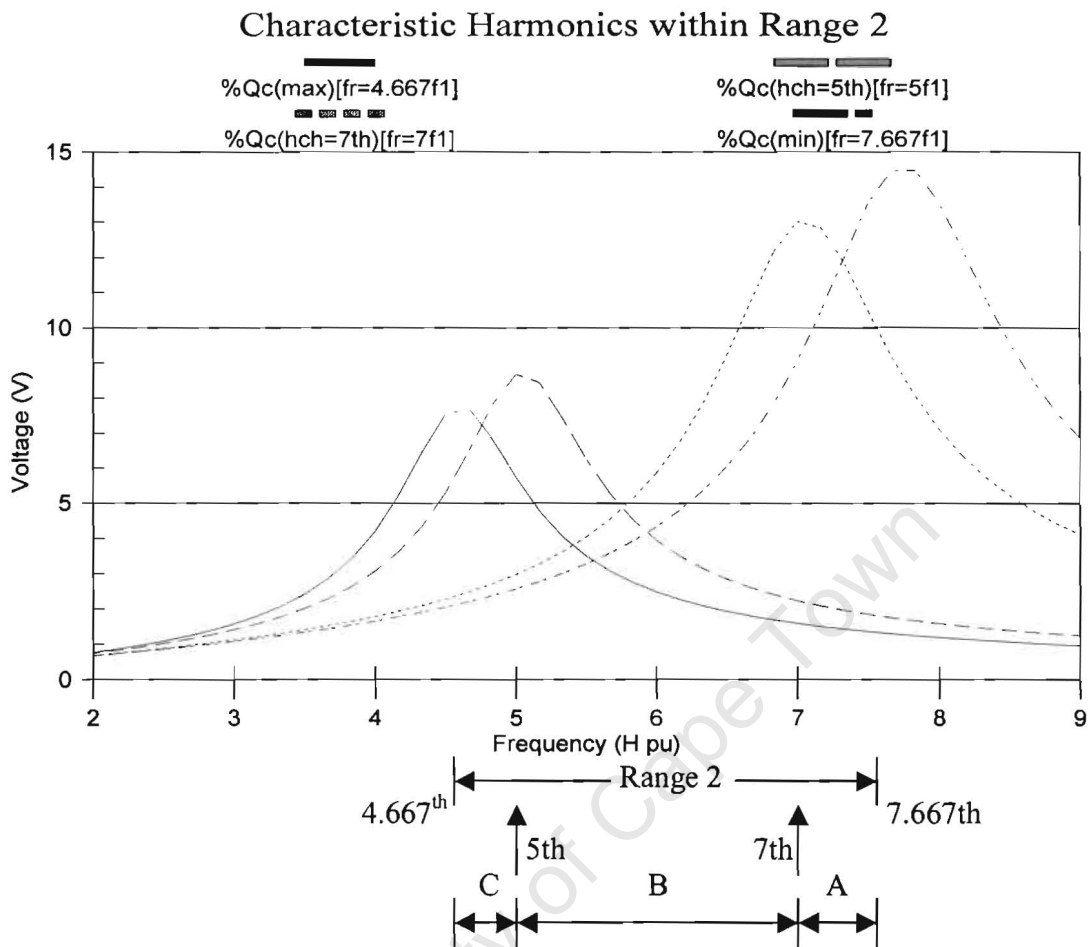


Figure 5.8 Sub-regions A, B and C within Range 2

As capacitor values have tolerances, %Q_C values should be chosen to prevent overlapping of curves.

Thus midpoint (mpt) values within each sub-region should be chosen.

This provides some robustness to the process, allowing for resonance changes due to small changes in system configuration, for example, new lines and/or equipment being added.

The midpoint values within each sub-region and the four main curves are identified as the decision alternatives that are required to investigate the problem, they are:

CONTROLLABLE INPUT 1	
DECISION	CHOICE VALUE FOR %Q _{C(ω1)}
a ₁	%Q _{C(MIN)}
a ₂	%Q _{C(mpt-A)}
a ₃	%Q _{C(7th)}
a ₄	%Q _{C(mpt-B)}
a ₅	%Q _{C(5)}
a ₆	%Q _{C(mpt-C)}
a ₇	%Q _{C(MAX)}

Table 5.4 Choice value for %Q_C

Thus “seven” possible decision alternatives (controllable input 1) are identified for investigating the problem.

STEP 6:

Calculate the %Q_{C(an)} values for the approximate mid points a₂, a₄ and a₆. Then using the power triangle and the calculated %Q_{C(an)} values determine their dpf 's. The %Q_{C(an)} values have already been calculated for a₁, a₃, a₅ and a₇, namely:

$$a_1 = \%Q_{C(MIN)} = 956.44 \text{ kvars (0.916 dpf)}$$

$$a_3 = \%Q_{C(7)} = 1157.20 \text{ kvars (0.933 dpf)}$$

$$a_5 = \%Q_{C(5)} = 2246.00 \text{ kvars (0.993 dpf)}$$

$$a_7 = \%Q_{C(MAX)} = 2700.00 \text{ kvars (unity dpf)}$$

The Q_C, upper and lower limits for each sub-region are known. As the line between two apex limits is a straight line, the approximate midpoint Q_C values are as follows:

$$\text{Midpoint}_{(\text{SUB-REGION})} = Q_{C(\text{HIGHEST VALUE OF SUB-REGION})} - [(\Delta Q_C) \div 2] \quad (5.17)$$

$$\text{where: } \Delta Q_C = Q_{C(\text{HIGHEST VALUE OF SUB-REGION})} - Q_{C(\text{LOWEST VALUE OF SUB-REGION})}$$

therefore:

$$a_2 = Q_{C(\text{mpt-A})} = 1157.20 - [(1157.20 - 956.44) \div 2] = 1056.8 \text{ kvars (0.925 dpf)}$$

$$a_4 = Q_{C(\text{mpt-B})} = 2246.00 - [(2246.00 - 1157.20) \div 2] = 1701.6 \text{ kvars (0.970 dpf)}$$

$$a_6 = Q_{C(\text{mpt-C})} = 2700.00 - [(2700.00 - 2246.00) \div 2] = 2473.0 \text{ kvars (0.998 dpf)}.$$

Set the capacitor value in the end-user computer model, in turn, to these seven Q_C values and plot their curves with scan studies, namely (figure 5.9):

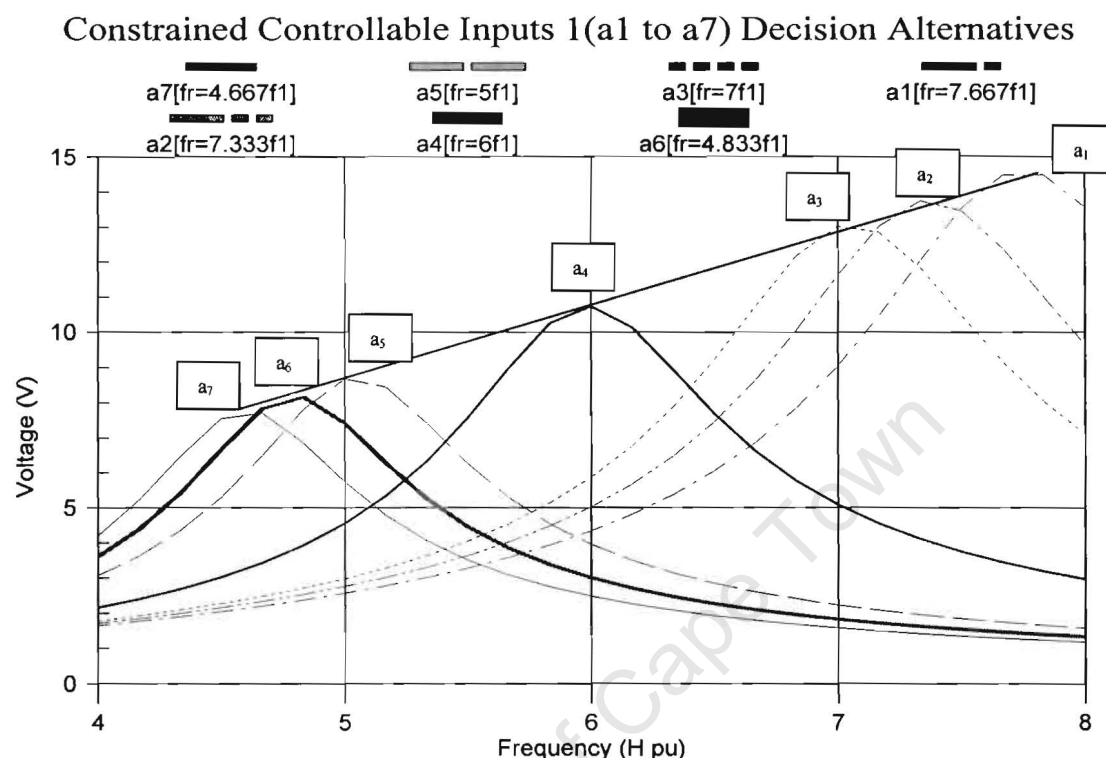


Figure 5.9 Controllable inputs 1

STEP 7:

Identify, the minimum number of controllable input 1, alternatives needed to give a solution to the problem. The decision-maker can proceed and investigate the severity of resonance for all the seven alternatives and more if he so chooses. However, decision-making is about “*choice*” and about meaningful and feasible solutions. It is a recommended principle of decision analysis that the least amount of choice is better and the choice is best if limited to only two controllable input 1 alternatives. As the effects of resonance can be damaging, any $\%Q_C$ alternative that resonates at a characteristic harmonic is not a likely choice, thus $a_{3(7th)}$ and $a_{5(5th)}$ can be eliminated.

It is traditional to improve a power factor to better than $\%Q_{C(MIN)}$ and not to correct to $\%Q_{C(MAX)}$. Therefore a_1 and a_7 can be eliminated from the choices. The decision-maker is therefore left with the three midpoint choices, a_2 , a_4 and a_6 . As $\%Q_{C(mpt-C)}$ has a $d_{pf} = 0.998$ and is close to unity d_{pf} correction, therefore “ a_6 ” can also be eliminated, leaving the decision-maker with only two controllable input 1 alternatives, a_2 and a_4 to investigate the severity of harmonic resonance.

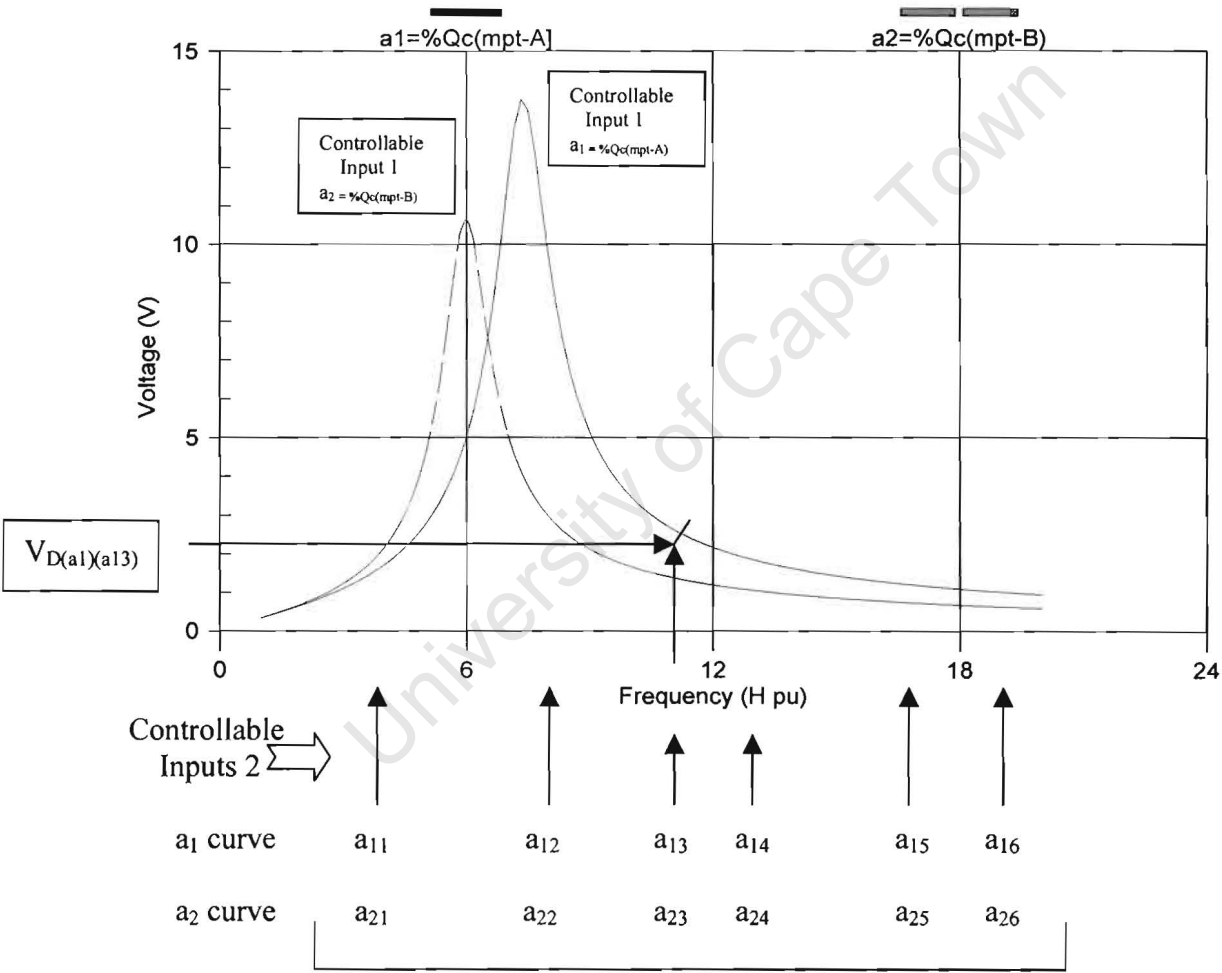
Note: For convenience let a_2 and a_4 be re-numbered as a_1 and a_2 respectively, thus:

CONSTRAINED CONTROLLABLE INPUT 1	
DECISION	CHOICE VALUE FOR % $Q_{C(\omega_1)}$
a_1	% $Q_{C(mpt-A)}$
a_2	% $Q_{C(mpt-B)}$

Table 5.5 Constrained controllable input 1

STEP 8:

Constrained Controllable Input 1 to a_1 and a_2



Characteristic harmonic injections from drive 1

Figure 5.10 Constrained Controllable inputs 1 and 2

As shown earlier, the CHRFB falls within the RFB and therefore the CHRFB is constrained to $6k \pm 1$, $k = 1, 2$, and 3 harmonics only. Thus, only the 5^{th} , 7^{th} , 11^{th} , 13^{th} , 17^{th}

and 19th harmonics need to be considered in relation to a_1 and a_2 . To visualize their influence on a_1 and a_2 , set the capacitor value in the end-user computer model, in turn, to the a_1 and a_2 value, respectively and conduct a “Harmonic Scan” study to derive their resonant curves. Superimpose the two curves together on the same axes (figure 5.10).

To visualize the influences of the injected harmonics, determine from the two curves the driving point voltages $V_{D(hch)}$ for the hch values = $6k \pm 1$, $k = 1, 2$ and 3 harmonics. For the a_1 curve, the six characteristic harmonics, a_{11} to a_{16} give rise to six driving point voltages $V_{D(hch)}$, namely:

a ₁ CURVE		
hch	V _{D(hch)}	VALUE (V)
a ₁₁	V _D (5)	2.756
a ₁₂	V _D (7)	11.615
a ₁₃	V _D (11)	2.655
a ₁₄	V _D (13)	1.842
a ₁₅	V _D (17)	1.183
a ₁₆	V _D (19)	1.012

Table 5.6 a₁ Curve

Similarly, for the a_2 curve,

a ₂ CURVE		
hch	V _{D(hch)}	VALUE (V)
a ₂₁	V _D (5)	4.550
a ₂₂	V _D (7)	5.088
a ₂₃	V _D (11)	1.388
a ₂₄	V _D (13)	1.052
a ₂₅	V _D (17)	0.725
a ₂₆	V _D (19)	0.613

Table 5.7 a₂ Curve

Therefore, associated with each controllable input 1, a_1 and a_2 , there are six controllable input 2 alternatives, namely, 5th, 7th, 11th, 13th, 17th and 19th harmonics, respectively.

Thus, when developing the mathematical model, for our case study, the decision maker must not only constrain the model to a_1 and a_2 but ensure that the objective function for the model reflects the range of the characteristic harmonics (a_{11} to a_{16} and a_{21} to a_{26}) as well. Table 5.8, therefore identifies the decision alternatives that should be considered when formulating the objective function for the case investigated.

DECISION ALTERNATIVES FOR CASE 1	
CONTROLLABLE INPUT 1	CONTROLLABLE INPUT 2
a_1	a_{11}
	a_{12}
	a_{13}
	a_{14}
	a_{15}
	a_{16}
a_2	a_{21}
	a_{22}
	a_{23}
	a_{24}
	a_{25}
	a_{26}

Table 5.8 Identification of decision alternatives for case 1

5.3.5 DECISION TABLE AND OBJECTIVE FUNCTION TO STRUCTURE AND REPRESENT THE HARMONIC RESONANCE DECISION PROBLEM (BLOCK D)

For our case study, taking constraints into account, the decision table is as follows:

SEVERITY OF HARMONIC RESONANCE DECISION TABLE				
DECISION ALTERNATIVES		STATES OF NATURE		
CONTROLLABLE INPUTS		UNCONTROLLABLE INPUTS		
1	2	s_1 = minimum demand	s_2 = mean demand	s_3 = full load demand
a_1 $= \%Q_{C(mpt-A)}$	$a_{11} = 5^{th}$	$HRSI_{(s1)(a11)}$	$HRSI_{(s2)(a11)}$	$HRSI_{(s3)(a11)}$
	$a_{12} = 7^{th}$.	.	.
	$a_{13} = 11^{th}$.	.	.
	$a_{14} = 13^{th}$.	.	.
	$a_{15} = 17^{th}$.	.	.
	$a_{16} = 19^{th}$	$HRSI_{(s1)(a16)}$	$HRSI_{(s2)(a16)}$	$HRSI_{(s3)(a16)}$
a_2 $= \%Q_{C(mpt-B)}$	$a_{21} = 5^{th}$	$HRSI_{(s1)(a21)}$	$HRSI_{(s2)(a21)}$	$HRSI_{(s3)(a21)}$
	$a_{22} = 7^{th}$.	.	.
	$a_{23} = 11^{th}$.	.	.
	$a_{24} = 13^{th}$.	.	.
	$a_{25} = 17^{th}$.	.	.
	$a_{26} = 19^{th}$	$HRSI_{(s1)(a26)}$	$HRSI_{(s2)(a26)}$	$HRSI_{(s3)(a26)}$
PROBABILITIES		p_{s1}	p_{s2}	p_{s3}

Table 5.9 Severity of harmonic resonance – Decision Table.

CHAPTER 5

Formulate the objective function for the Harmonic Resonance Severity Index (%HRSI): (chapter 4, equation 4.22).

$$\%HRSI_{(sN)(anm)} = \frac{V_{C(sN)(anm)} [\%Q_{C(an)} (fr \neq hch)]}{V_{C(sN)(anm)} [\%Q_{C(an)} (fr = hch)]} \times 100\% \quad (5.18)$$

subject to: $hch = 6k \pm 1$, $k = 1, 2$ or 3 harmonics only

$anm = hch$

$\%Q_{C(MIN)} < \%Q_{C(an)(fr = hch)} < \%Q_{C(MAX)}$

$\%Q_{C(MIN)} = \%Q_{C(dpf = 0.9)}$

$\%Q_{C(MAX)} = \%Q_{C(dpf = 1)}$

$\%Q_{C(MIN)} \leq \%Q_{C(an)} (fr \neq hch) \leq \%Q_{C(MAX)}$

$\%Q_{C(a1)(fr \neq hch)} = \%Q_{C(mpt - A)}$

$\%Q_{C(a2)(fr \neq hch)} = \%Q_{C(mpt - B)}$

$\%Q_{C(mpt - A)} = \text{midpoint between } \%Q_{C(MIN)} \text{ and } \%Q_{C(7)(fr = 7)}$

$\%Q_{C(mpt - B)} = \text{midpoint between } \%Q_{C(7)(fr = 7)} \text{ and } \%Q_{C(5)(fr = 5)}$

$s_N = s_1, s_2$ or s_3 only.

$s_1 = (0.25)$ full load demand (capacitor set to full load (ω_1) value),

$I_{l(\omega_1)}$ and $I_{(hch)}$ values of harmonic producing load and the linear load in the plant are adjusted (0.25FL) accordingly.

$s_2 = (0.6)$ full load demand (capacitor bank size remains fixed to full load (ω_1) value, $I_{l(\omega_1)}$ and $I_{(hch)}$ values of harmonic producing load and the linear load in the plant are adjusted (0.6FL) accordingly.

$s_3 = (1.0)$ full load demand (capacitor bank size remains fixed to full load (ω_1) value), $I_{l(\omega_1)}$ and $I_{(hch)}$ values of harmonic producing load and the linear load in the plant are adjusted (1.0FL) accordingly.

Calculate the $\%HRSI_{(sN)(anm)}$ values for the decision table. The results of the calculations are given in Appendix 4. The results are substituted in the decision table.

CHAPTER 5

SEVERITY OF HARMONIC RESONANCE — DECISION TABLE				
DECISION ALTERNATIVES		STATES OF NATURE		
CONTROLLABLE INPUTS		UNCONTROLLABLE INPUTS		
1	2	s_1 = minimum demand	s_2 = mean demand	s_3 = full load demand
a_1 $= \%Q_{C(mpt-A)}$	$a_{11} = 5^{th}$	37.73%	33.04%	31.09%
	$a_{12} = 7^{th}$	118.89%	102.95%	89.11%
	$a_{13} = 11^{th}$	12.75%	11.95%	12.21%
	$a_{14} = 13^{th}$	7.68%	7.05%	7.03%
	$a_{15} = 17^{th}$	3.84%	3.45%	3.69%
	$a_{16} = 19^{th}$	2.94%	2.63%	2.55%
a_2 $= \%Q_{C(mpt-B)}$	$a_{21} = 5^{th}$	82.69%	66.93%	58.42%
	$a_{22} = 7^{th}$	29.98%	30.01%	33.03%
	$a_{23} = 11^{th}$	6.43%	5.92%	5.92%
	$a_{24} = 13^{th}$	4.20%	3.81%	3.76%
	$a_{25} = 17^{th}$	2.24%	2.00%	1.94%
	$a_{26} = 19^{th}$	1.75%	1.56%	1.50%
PROBABILITIES		$P_{s1} = 0.3$	$P_{s2} = 0.6$	$P_{s3} = 0.1$

Table 5.10 Decision table for case 1

One of the objectives [section 5.3.2 (b)], is to make a decision if the severity falls within the severe and very severe categories of harmonic resonance.

On screening the decision table, it can be seen that a_{11} , a_{13} to a_{16} and a_{22} to a_{26} fall in the category of “least severity of resonance” (table 4.3) and can be eliminated.

The “Relevant Decision Table” is thus:

SEVERITY OF HARMONIC RESONANCE — DECISION TABLE				
DECISION ALTERNATIVES		STATES OF NATURE		
CONTROLLABLE INPUTS		UNCONTROLLABLE INPUTS		
1	2	s_1 = minimum demand	s_2 = mean demand	s_3 = full load demand
a_1 $= \%Q_{C(mpt-A)}$	$a_{12} = 7^{th}$	118.89%	102.95%	89.11%
a_2 $= \%Q_{C(mpt-B)}$	$a_{21} = 5^{th}$	82.69%	66.93%	58.42%
PROBABILITIES		$P_{s1} = 0.3$	$P_{s2} = 0.6$	$P_{s3} = 0.1$

Table 5.11 Relevant decision table for case 1

This brings us to the end of stage 1 of the decision theory process for making a decision on the severity of harmonic resonance. The decision table thus structures and represents

the decision problem on harmonic resonance and takes into account all the main ingredients of the decision model.

5.4 STAGE 2 – DECISION MAKING: DECISION 1 (BLOCK E)

Utility theory assumes that every decision-maker uses a utility function that translates each of the possible outcomes in a decision problem into a measure of true worth allocated by the decision-maker. This worth is known as a utility.

The advantage of utility theory is that it enables risk averse, risk seeking and risk neutral (risk indifferent) decision-maker's to accommodate personal desire abilities or preferences and make a rational decision (figure 3.15).

Another advantage of utility theory is that it can be applied to outcomes, which have non-monetary values (e.g., outcomes expressed in percentages, such as %HRSI) (chapter 3, section 3.19).

Decision-maker's concerned with the severity of harmonic resonance are more than likely to be risk averse people. Utility theory needs to be applied to select the decision alternative that best meets the objective.

Following the utility theory method developed in chapter 4, section 4.9(a) to (f), we proceed as follows:

- a. Based on Table 5.11, rank the outcomes from best (low %HRSI) to worst (high %HRSI), namely:

58.42% (best)
66.93%
82.69%
89.11%
102.95%
118.89% (worst)

- b. Assign utility values to the best and worst outcomes. Let us assign a utility value of "0" to 118.89% and a utility value of "1" to 58.42%.
- c. Next we derive the utility function for the decision-maker. A risk averse descending utility function is expected. Plot the risk neutral utility function (see figure 5.11 below) as the initial step towards deriving a concave curve.
 - (i) Apply the variable probability method to elicit probability indifference (p_{ind}) values for the other outcomes, using an interview to conduct an elicitation session, namely:

CHAPTER 5

For the %HRSI = 66.93% outcome:-Question to the decision-maker: “which p value would make you indifferent between A and B”?

A: 66.93% for certain

B: a p chance of obtaining 58.42% (best outcome), or a 1-p chance of obtaining 118.89% (worst outcome).

If $p = 0.5$ would you be indifferent? Answer: No, I would not be indifferent, I would choose “A” because “B” would only give me a 50% chance of obtaining the best outcome, but it would also give me a 50% chance of obtaining the worst outcome.

The situation must therefore be made more attractive by increasing “p”. Use the risk neutral curve as a guideline for attractiveness and it can be seen that 66.93% has a risk neutral utility value of 0.85. Thus, if $p = 0.9$ would you be indifferent? Answer: Yes I would be indifferent. Both A and B would be equally attractive. “A” would give me 66.93% severity for certain (for sure) and B would give a high chance for obtaining the best outcome and only $1 - 0.9 = 0.1$ (10%) chance of obtaining the worst outcome. The risk is only 10%. If $p > 0.9$ say 0.95, I would not be indifferent and would choose B as there would be a 95% chance of obtaining the best and only a 5% chance for the worst and this would lead to a result better than 66.93%.

Therefore, $p_{ind} = 0.9$ for 66.93% outcome.

For the %HRSI = 82.69% outcome:-Following a similar elicitation session as for 66.93% outcome (e.g., $p = 0.5$ etc.), the situation is made more attractive until the answer is yes.

There is a rule of utility, which states that the more desirable an outcome the higher the utility and therefore, vice versa. The less desirable an outcome the lower is the utility value. As 82.69% is less desirable than 66.93 it is expected that the “ p_{ind} ” value will also be less. The risk neutral plot suggests 0.6. After a series of questions, the decision-maker is indifferent if $p = 0.8$.

Therefore, $p_{ind} = 0.8$ for 82.69% outcome.

Similarly, for %HRSI = 89.11% and 102.95%, the decision-maker is indifferent when $p = 0.7$ and 0.6, respectively.

Thus: $p_{ind} = 0.7$ for 82.69%
 $p_{ind} = 0.6$ for 102.95%

(ii) Calculate the utility values and plot the utility function: (chapter 3, equation 3.19)

$$U(\%HRSI)(\text{sure outcome}) = p_{ind} U(\%HRSI)(\text{best outcome}) + (1 - p_{ind}) U(\%HRSI)(\text{worst outcome}) \quad (5.19)$$

As, “1” and “0” have been allocated to best and worst outcomes, respectively:

$$U(\% \text{HRSI})(\text{sure outcome}) = p_{\text{ind}} U(\% \text{HRSI})(\text{best outcome}) \quad (5.20)$$

Note: Once, the p_{ind} has been elicited, the utility value is simply equal to the p_{ind} value.

$U(58.42\%) = 1.0$
$U(66.93\%) = 0.9$
$U(82.69\%) = 0.8$
$U(89.11\%) = 0.7$
$U(102.95\%) = 0.6$
$U(118.89\%) = 0.0$

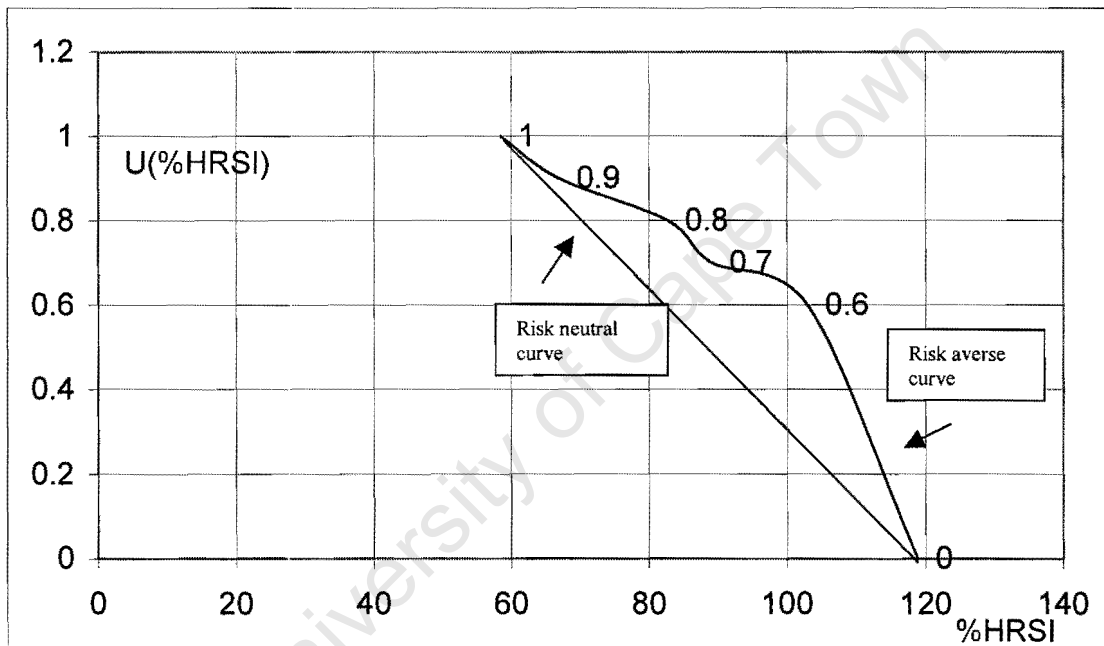


Figure 5.11 Utility function for case 1

The graph (figure 5.11) is an estimate of the decision-makers utility function for this decision problem. Note the shape of the utility function is concave (risk averse).

Note: To derive a utility function for a decision-maker for a specific decision problem, it is not necessary to use the outcomes from the decision table; one can hypothesize using any severity outcomes provided they fall within the best and worst outcomes of the decision problem. Once the utility function has been estimated, then the outcomes on the decision table can be applied to the curve and the utility values can be read off.

(iii) Next develop the utility table.

UTILITY TABLE FOR CASE 1 (RISK AVERSE)				
DECISION ALTERNATIVES		STATES OF NATURE		
CONTROLLABLE INPUTS		UNCONTROLLABLE INPUTS		
1	2	s ₁	s ₂	s ₃
a ₁	a ₁₂ = 7 th	0.0	0.6	0.7
a ₂	a ₂₁ = 5 th	0.8	0.9	1.0
PROBABILITIES		P _{s1} = 0.3	P _{s2} = 0.6	P _{s3} = 0.1

Table 5.12 Utility table for case 1 (risk averse)

The %HRSI outcomes on the decision table (table 5.11) are replaced in the utility table (table 5.12) with utility values.

- d. Calculate the expected utility value for each decision alternative: (chapter 3, equation 3.13).

$$EU_{(an)(anm)} = \sum_{N=1} p_{(sN)} U[\%HRSI_{(sN)(anm)}] \quad (5.21)$$

namely:

$$EU_{(a1)(a12)} = (0.3)(0) + (0.6)(0.6) + (0.1)(0.7) = 0.43 \text{ lowest}$$

$$EU_{(a2)(a21)} = (0.3)(0.8) + (0.6)(0.9) + (0.1)(1.0) = 0.88 \text{ highest}$$

- e. Select the highest $EU_{(an)(anm)}$ value and this gives the size of the capacitor % $Q_{C(an)(anm)}$ to be installed.

The highest $EU_{(an)(anm)}$ value is 0.88, thus % $Q_{C(a2)(a21)}$ is the size to be installed.

- f. For this size, make decision 1 and determine from the decision table the severity of harmonic resonance for the states of nature, thus: (chapter 4, table 4.7).

DECISION 1

DECISION 1 IN THE PROCESS OF MAKING A MITIGATION DECISION					
Decision alternatives		States of nature			Expected utility Value
Controllable inputs		Uncontrollable inputs			
1	2	s ₁	s ₂	s ₃	EU
a _n =%Q _{C(an)} =1.7016 Mvars (selected capacitor size)	a _{nm} = 5 th (m th harmonic)	%HRSI _{(s1)(anm)} = 82.69%	%HRSI _{(s2)(anm)} = 66.93%	%HRSI _{(s3)(anm)} = 58.42%	EU _{(an)(anm)} =0.88 (highest value)

Table 5.13 Mitigation decision 1 for case 1

The capacitor size to be installed is 1.7016Mvars (0.9702 dpf). This capacitor will cause 82.69%, 66.93% and 58.42% severities of harmonic resonance, respectively when the plant operates under the three states of nature.

The developed utility theory method thus supports a rational decision and shows that a_2 is preferred to a_1 (The severities for a_2 are lower than those of a_1). Decision 1 is therefore based on a scientific theory.

5.5 STAGE 3 – MAKING A MITIGATION DECISION ON HARMONIC RESONANCE: DECISION 2 (BLOCK F)

The next question is: “If this size of capacitor is installed, will the severities of harmonic resonance cause damage to the capacitor under the three states of nature”? Despite the severities being less than 100% it is good practice to check if mitigation is needed or not.

To make decision 2, we need to apply the newly developed formula as given in equation 4.25:

$$\%MI_{\%HRSI(sN)(anm)} = \frac{I_{C(RMS)}}{1.30I_1} \times 100\% \quad (5.22)$$

$$\text{subject to:} \quad V_{C1} \leq U_N \quad (5.23)$$

Calculate rated current and voltage at the fundamental frequency:

$$U_N = V_{LINE}/\sqrt{3} = 4.16 \times 10^3/\sqrt{3} = 2401.77V$$

$$Q_C \text{ rated} = \sqrt{3}V_{LL}, \therefore I_L = 1.7016 \times 10^6/\sqrt{3}(4.16 \times 10^3) = 236.158A, \text{ thus } I_1 = 236.158A$$

Carry out a harmonic penetration study for the three states of nature (s_1, s_2 and s_3) and calculate $I_{C(rms)}$ and V_{C1} for the capacitor size (1.7016Mvars) resulting in the severities of harmonic resonance as per the decision table, (table 5.11). The results of the studies are given in Appendix 5, section 5.1 (RMS results = $I_{C(RMS)}$ values and the V_{C1} are found under magnitude).

Calculate the mitigation index values reflecting payoff outcomes. The calculations are shown in Appendix 5, section 5.2. The calculated values are listed in table 5.14. Decision 2 can thus be made:

DECISION 2

DECISION 2 – MAKING A MITIGATION DECISION				
VARIABLES				
“sN”	“anm”	%HRSI	%MI	%V _{C1}
s ₁	5th	82.69%	80.52%	101.00%
s ₂		66.93%	86.84%	97.66%
s ₃		58.42%	96.48%	93.95%

Table 5.14 Mitigation decision 2 for case 1

The results in table 5.14 show that no mitigation is needed since $\%MI < 100\%$ and $V_{C1} < 1.10U_N$. After installation the capacitor will therefore not be damaged and can be installed with confidence.

CHECKING:

The IEEE provides standard loading indices for pf capacitors. (see Appendix 1). [3]

The IEEE limits (Appendix 1) were applied to check decision 2, that is, that the capacitor will not suffer damage. The results of these checks can be seen in Appendix 5, section 5.3.

They confirm the finding that no mitigation device (e.g., a passive filter) needs to be installed in the end-user plant to safeguard the capacitor.

5.6 SUMMARY

On application of the three-stage decision theory process, it is found that no mitigation remedy is needed for case 1. The capacitor size chosen by the process can be installed with confidence.

The structured approach to making a mitigation decision, using the principles of a decision table, (stage 1), utility theory (stage 2) and making a mitigation decision (stage 3) has proven to be effective.

From the test system conducted in case 1, decision theory can be used for making a harmonic resonance mitigation decision and has provided a foundation for its application to more complex case studies as discussed in the next chapters.

CHAPTER 6

CASE STUDY 2

APPLICATION OF DEVELOPED DECISION THEORY PROCESS TO A POWER SYSTEM HAVING MULTIPLE RESONANT POINTS

In this chapter, the developed three-stage decision theory process is applied to a power system containing an interconnection of capacitances and inductances, and three end-users are supplied. Some recordings of field measurements are available. The measurements show that the system is experiencing harmonic resonance. One end-user has a very low power factor and wants to install a power factor correction capacitor. Computer models are developed and it is shown that the simulation results are similar to the measurements for the worst case-operating scenario. Each of the three stages of the developed decision theory process is applied and details of each step in the process are given. Multiple harmonic resonance points are found. An objective function is formulated taking into account the constraints relevant to case 2. The %HRSI_(sN) outcomes are calculated and a relevant decision table is developed. A utility function is developed for the decision-maker and a risk averse utility table is derived. Decision 1 is made and the capacitor size which best meets the objective is chosen. %MI indices are calculated for the states of nature and a mitigation decision (decision 2) is made. Other mitigation concerns, not directly relevant to the topic researched, are discussed for completeness.

6.1 DESCRIPTION OF CASE 2

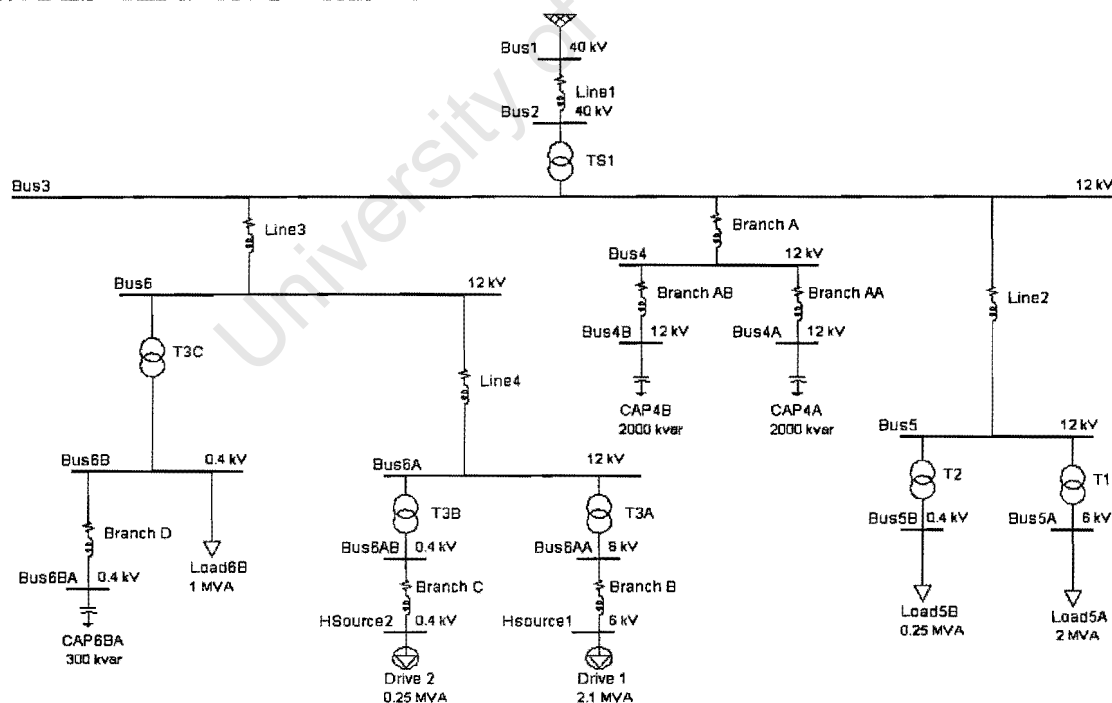


Figure 6.1 One-line-diagram – Case 2

Figure 6.1 shows the one-line-diagram used for this investigation. This system was taken from reference [1].

The system is supplied from 40kV, which at bus 3 (12kV) is feeding two distribution networks. At bus 4 there are two capacitor banks and at bus 6BA there is one capacitor bank connected for power factor correction. There are two harmonic sources (6-pulse drives) at buses 6AA and 6AB, respectively. Each harmonic source is operating at a different voltage level, 6kV and 0.4kV, respectively. Each of the drives inject only $h_{ch} = 6k \pm 1$, $k = 1, 2$ and 3 characteristic harmonics into the system.

The system in figure 6.1 is a representation of a real system as the results disclosed in the paper are based on field measurements [1]. The one-line-diagram has been re-drawn and labeled in preparation for the development of the computer models to be used for the decision investigation.

6.1.1 ESTABLISHING PARAMETER VALUES FOR COMPUTER MODELS

The results disclosed [1] show that when all capacitors are in operation, a resonance exists at or close to the 5th harmonic. The 5th harmonic voltage and current are 12.50% and 30A, respectively.

Before proceeding with the application of the decision theory process, it was necessary to conduct a computer study of the system (figure 6.1). The results obtained would show that they are similar to those disclosed in paper [1] and in this way, establish that the modeling employed in this thesis is correct (benchmark). A scan model and a harmonic penetration model are employed to evaluate the field measurement results obtained in the paper [1].

The computer models for conducting scan and penetration studies are given in Appendix 6, sections 6.1 to 6.3. The programs were run and the voltage and current results (Appendix 6, section 6.3.1 and 6.3.2,) obtained were then compared to the field measurements. The scan result is shown in figure 6.2:

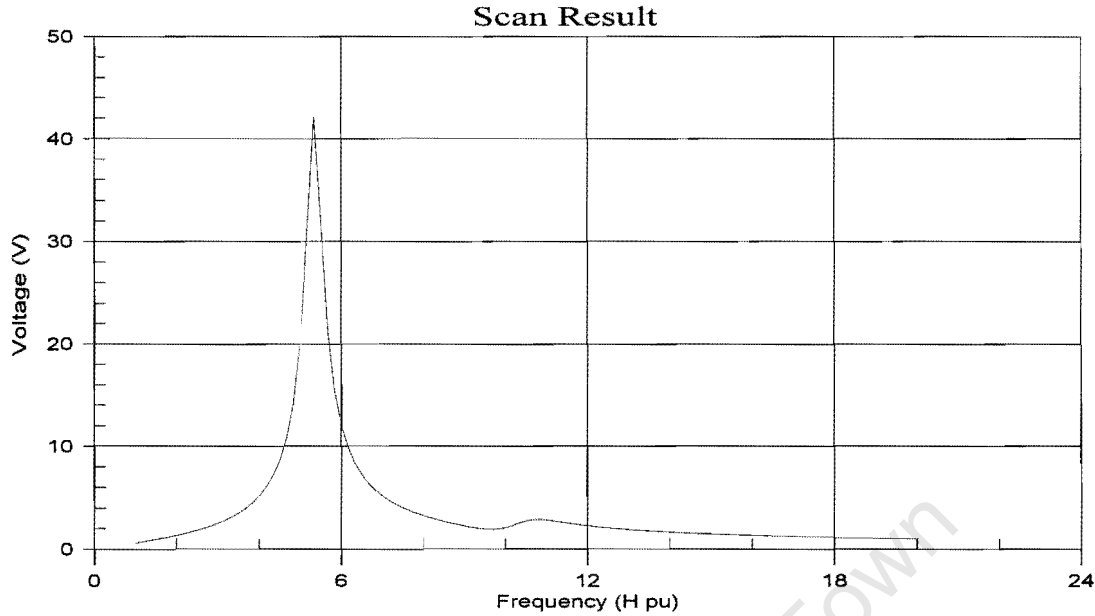


Figure 6.2 Resonance close to or at 5th harmonic

The scan shows a resonance close to the 5th harmonic and this confirms the result disclosed in [1].

The harmonic penetration results are:

a) 5th harmonic phase voltage: 791.707V.

The line voltage at bus 6A where the measurements were taken is 12kV, thus, $(791.707/6928.2) \times 100\% = 11.42\%$ compared to 12.50% measured voltage.

b) 5th harmonic current = 26.75A compared to 30A measured (Line 3).

The difference in results is because transformer details are not disclosed in [1] and values had to be assumed for them.

As the results confirm the measurements, the scan and penetration computer models developed for the system can be used for further investigations [benchmark case].

6.2 DECISION THEORY PROCESS

The capacitor banks at buses 4A, 4B and 6BA are already installed and their Mvar sizes have been pre-determined and are therefore fixed [1].

However, from [1], end-user 1 (bus 5A) has a $\text{dpf} = 0.5$ lagging which is very low. This is one of the main reasons for choosing this system for investigation. End-user 2 (bus 5B)

has a 0.9 dpf and the load at bus 6B (part of end-user 3 network) has a $\text{dpf} = 0.936$. The latter two end-users are therefore assumed not to require any further pf correction.

Using results from the benchmark case (Appendix 6, section 6.4), it can be confirmed that end-user 1 operates at a dpf of 0.5 lagging. The voltage and current at “ ω_1 ” at bus 5A are:

$$3322.59 \angle -2.9^\circ \text{V and } 184.58 \angle -62.9^\circ \text{A}$$

thus:

$$\begin{aligned} \text{phase angle} &= -62.9^\circ - (-2.9^\circ) = -60^\circ \\ \text{dpf} &= \cos 60^\circ = 0.5 \text{ lagging} \end{aligned}$$

As the system contains capacitors and two harmonic sources it has an existing resonance point close to or at the 5th harmonic ($f_r = 5.333f_1$).

This is different from case 1 which did not contain any capacitors, therefore there was no pre-existing resonance point. It also only had one harmonic source and had only one consumer. In this present case, three end-users are identified, bus 5A (end-user 1, no harmonic source), bus 5B (end-user 2, no harmonic source) and bus 6 (end-user 3, two harmonic sources, one at bus 6AB and the other at bus 6AA besides a load and capacitor bank at bus 6B). There are also capacitor banks connected to the point of common coupling (PCC) at bus 3.

6.2.1 DECISION SCENARIO

The following decision scenario needs investigation.

End-user 1 wants to introduce a pf correction capacitor bank to his plant to improve his low dpf of 0.5 lagging. He is aware that together with the other capacitors in the system and the two 6-pulse drives in end-user 3’s plant, the existing resonance point ($f_r = 5.333$) will change. In addition, the size of the capacitor bank chosen could result in damage from the severity of harmonic resonance and there may be a need for a mitigation device to be installed in his plant. He is also aware that the level of power drawn by his plant impacts on the severity of harmonic resonance.

In addition, the end-user is also aware that he needs a decision-maker to make a decision on the size of capacitor to be installed and to make a decision as to whether or not mitigation is needed. [6]

6.3 DEFINE THE PROBLEM (BLOCK A)

The problem is, will the resonance be severe enough to damage the pf correction capacitor bank to be installed in end-user 1’s plant when the plant is operating at various levels of power drawn.

6.4 IDENTIFY THE OBJECTIVES (BLOCK A)

The decisions that need to be made are:

Decision 1:

Determine the severity of harmonic resonance caused by the installation of a pf correction capacitor bank at end-user 1's plant at key harmonic frequencies, for a given range of power demand (steady-state) operating conditions. That is, make a decision between different sizes of capacitors taking into account the preferences and desires of the decision-maker (decision 1).

Decision 2:

If severity levels of resonance are severe or very severe take a decision as to whether mitigation is needed or not (decision 2).

6.5 IDENTIFICATION OF STATES OF NATURE (BLOCK B)

A survey of harmonic voltages and currents was conducted at distribution substations in the USA in 1991. Recordings over 7 days of fundamental and higher harmonics were measured. Histograms were generated enabling the determination of maximum, mean and minimum levels of demand to be established [38].

The survey identifies three categories: minimum, mean and maximum demand levels as typical levels found in distributions systems.

The following statement further supports these categories [28]:

"The first step in designing any practical power factor correction scheme must be to obtain accurate details of the load conditions with values of kW, kVA and power factor at light, average and full load, together with the type and details of the loads".

If recordings of measurements are available for an end-user plant, histograms can be generated and states of nature identified for the minimum, mean and full load demand categories. If no recordings of measurements are available then the states of nature need to be estimated for the three categories of demand.

For the system in figure 6.1, no measurement or histogram data is disclosed in [1] for end-user 1.

Full load for end-user 1 is 2000 kVA (s_3 = full load demand). The other two categories are identified as 500kVA (s_1 = minimum demand) corresponding to 0.25FL and 1200kVA (s_2 = mean demand) corresponding to 0.6FL. Thus:

$$\begin{aligned}s_1 &= \text{minimum demand (0.25 FL)} \\ s_2 &= \text{mean demand (0.6 FL)} \\ s_3 &= \text{full load demand (1.0 FL)}\end{aligned}$$

Therefore, for the 2000kVA, 0.5 dpf load, the states of nature are:

$$S_{E(s1)} = P_{E(s1)} + j Q_{E(s1)} = 0.5 \angle -60^\circ = (0.25 + j 0.433) \text{MVA}$$

$$S_{E(s2)} = P_{E(s2)} + j Q_{E(s2)} = 1.2 \angle -60^\circ = (0.6 + j 1.03923) \text{MVA}$$

$$S_{E(s3)} = P_{E(s3)} + j Q_{E(s3)} = 2.0 \angle -60^\circ = (1 + j 1.73205) \text{MVA}$$

To work in the risk zone for decision making rather than in uncertainty, probability values $p_{(sN)}$ for the plant at minimum (p_{s1}) mean (p_{s2}) and full load (p_{s3}) need to be assigned to the states of nature. As subjective assessment of probabilities is required, the method of “relative heights” is used to assign probabilities to the three states of nature. This has already been done, see chapter 3, example 3.2, where the decision-maker has already assigned probabilities to the states of nature for the 2000kVA end-user plant, using the “relative heights”. The probabilities are therefore:

$$\begin{aligned}p_{s1} &= p(500) = 5/16 = 0.3 \\ p_{s2} &= p(1200) = 10/16 = 0.6 \\ p_{s3} &= p(2000) = 1/16 = 0.1\end{aligned}$$

Note: p_{sN} values have been rounded off to one decimal place.

As probabilities have been assigned to the states of nature, the decision model is deterministic and the decision-making is in risk zone.

6.6 IDENTIFICATION OF DECISION ALTERNATIVES (BLOCK C)

STEP1:

Identify the “RFB”. Start by determining the fundamental frequency power triangle for s_3 when no pf capacitor is installed at bus 5A. A pf correction capacitor is to be installed at bus 5AA in end-user 1’s plant (bus 5A).

Bus 5AA is not shown in figure 6.1 but is connected via a very low resistive path (branch E) to bus 5A. Bus 5AA is added so that the decision-maker can focus specifically on the new capacitor to be installed in the plant and so be separate from load 5A connected to bus 5A. Calculate the kvar values for the capacitor (CAP 5AA) step settings (% Q_C) to improve the dpf in steps from 0.5 to unity.

CAPACITOR BANK (CAP 5AA)		
STEP SETTING	dpf (lag)	Q _c (kvars)
0.000%	0.500	0.000
3.125%	0.512	54.127
6.250%	0.525	112.584
12.500%	0.550	216.507
25.000%	0.609	433.013
50.000%	0.756	866.025
75.000%	0.917	1299.037
100.000%	1.000	1732.050

Table 6.1 Capacitor bank step setting sizes

Conduct a harmonic scan study for each step setting and use “TOP” [37] to display their resonance curves (figure 6.3).

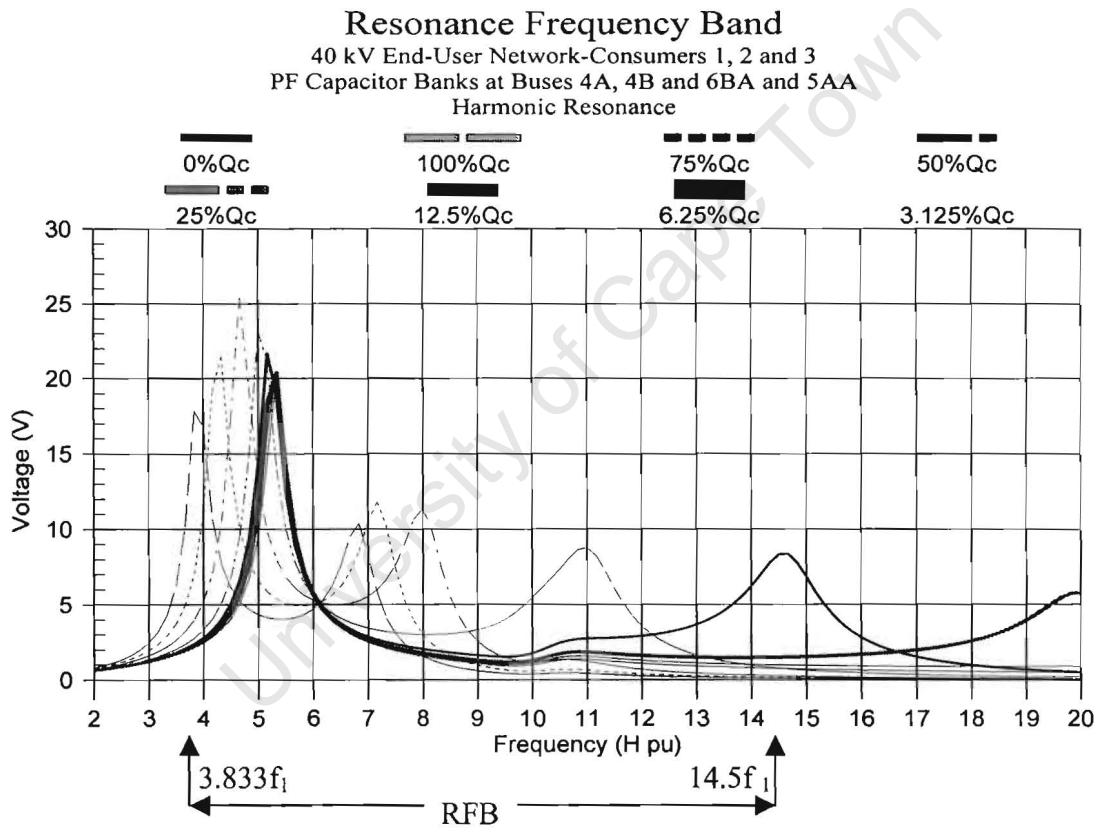


Figure 6.3 Resonance frequency band - case 2 (bus 5AA)

Note: The 0%Q_c step setting resonant point is hidden behind other curves but occurs at $f_r = 5.333$, remembering that the system has a pre-existing resonance point before the installation of a pf capacitor at bus 5AA.

The 3.125% step is a small change and the f_r remains close $5.333f_1$. For the 6.25% step there is also little change from the $5.333f_1$. However, at the 12.5% step, two resonance

CHAPTER 6

points are created, $5.167f_1$ and $14.5f_1$, respectively. At the 25% step, also two points, $5f_1$ and $11f_1$ are created. Similarly there is at 50%, one below the 5^{th} at $4.667f_1$ and the other above at $8f_1$. At the 75% step, there is one below the 5^{th} at $4.333f_1$ and one above at $7.167f_1$. At 100% step, there is also one below the 5^{th} at $3.833f_1$ and one above at $6.833f_1$.

Read off the co-ordinates of the apexes of the resonant points:

CO-ORDINATES OF RESONANCE POINTS FOR STEP SETTINGS			
%Q _C	fr(h)	V	dpf at bus 5A
100%	3.833	19.018	Unity pf
	6.833	11.061	
75%	4.333	22.912	0.917 lag
	7.167	12.610	
50%	4.667	27.136	0.756 lag
	8.000	12.150	
25%	5.000	24.724	0.609 lag
	11.000	9.358	
12.5%	5.167	23.143	0.5507 lag
	14.500	8.868	
6.25%	5.333	21.805	0.525 lag
	-	-	
3.125%	5.333	21.805	0.512 lag
	-	-	
0	5.333	21.805	0.500 lag
	-	-	

Table 6.2 Co-ordinates of resonance points for step settings

From the plots, determine the RFB and mark it on a plot diagram.

$$\text{RFB} \approx 3.833f_1 (100\%) \text{ to } 14.5f_1 (12.5\%Q_C) \quad (6.1)$$

The advantage of the RFB is that it delimits the resonance curves to a working range and is the initial step in the process for identifying the 2-controllable input decision alternatives.

Make an observation about the dpf values in table 6.2. For a 50% step the dpf improves from 0.5 to 0.756. For 75%, improves to 0.917, at 100% step, dpf is unity.

According to the rule of thumb used in industry the minimum dpf to which an end-user would improve to, is 0.9. (%Q_{C(MIN)}).

Calculate %Q_{C(MIN)} to operate the end-user at 0.9dpf. The value is 247.77 kvars. (72.04% correction). Carry out a scan study, then superimpose its curve together with the 100% step curve onto a plot diagram. See figure 6.4 below. Therefore, the controllable input 1, alternatives (size of capacitor) to be selected will fall within Range 1.

$$\text{Range 1} \approx 72.04\%Q_C \leq \%Q_C \leq 100\%Q_C \quad (6.2)$$

The co-ordinates are 100%[fr = 3.833, 19.018V] and 72.04% [fr = 4.333, 24.772V] for the first resonant apexes and 100%[fr = 6.833, 11.061V] and 72.039%[fr = 7.167, 12.448V] for the second resonant points, thus:

$RFB \approx 3.833f_1 (100\%) \text{ to } 7.167(72.04\%)$ (6.3)

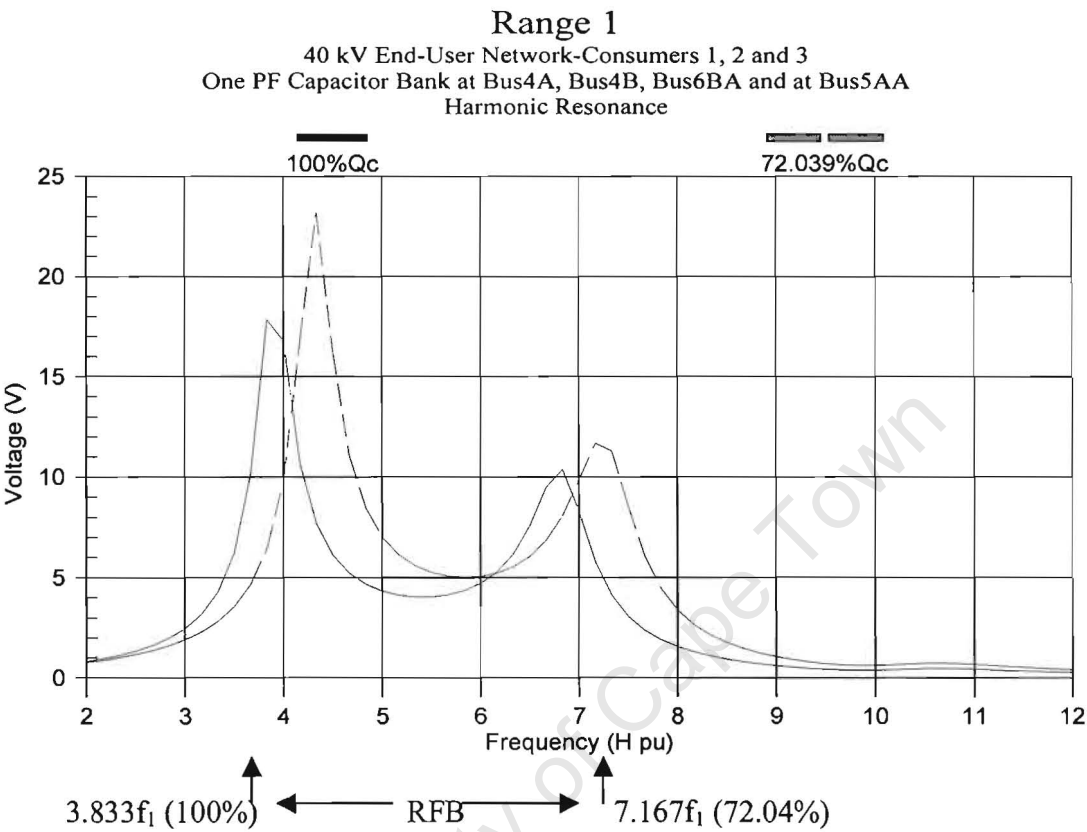


Figure 6.4 Range 1- case 2

STEP 2:

Identify the CHRFB. Only the 5th and 7th characteristic harmonics fall within Range 1, (hch = 6k±1, k = 1 only). Using the co-ordinates in table 6.2 and the straight line equation approach (case 1, chapter 5, equation 5.7), determine the %Q_C values required to tune the resonant point at bus 5AA to the 5th and 7th harmonic respectively, namely:

$Q_{C(5th)} = 433.010 \text{ kvars } (\%Q_{C(5th)}) = 25\% \text{ correction}$

$Q_{C(7th)} = 1399.037 \text{ kvars } (\%Q_{C(7th)}) = 80.774\% \text{ correction}$

Thus,

CAPACITOR SIZES IN RANGE 1	
Step setting (%Q _C)	dpf
100% (MAX)	1.000
80.774%(7 th)	0.948
72.039%(MIN)	0.900
25%(5 th)	0.609

Table 6.3 Capacitor sizes in Range 1

Superimpose their frequency scans on a common plot, namely (figure 6.5):

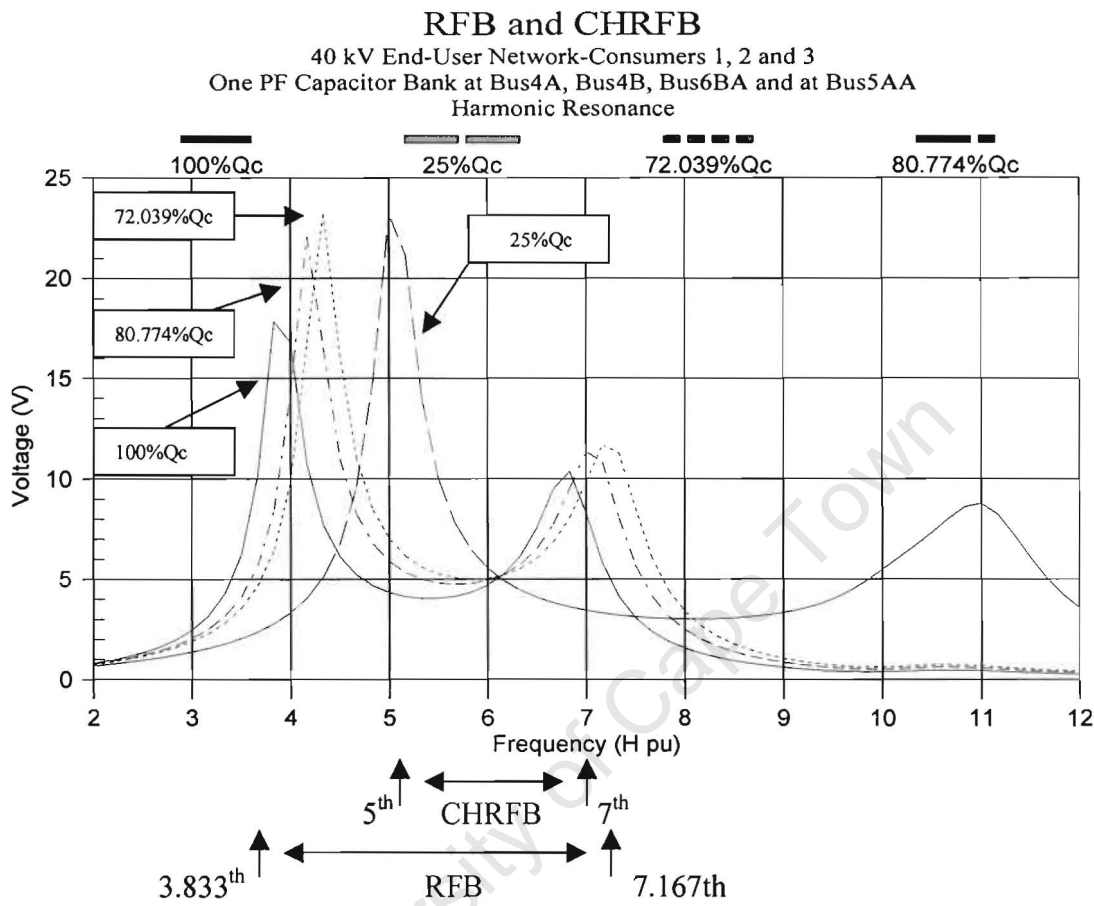


Figure 6.5 RFB and CHRFB - case 2

STEP 3:

Identify the controllable input 1 alternatives.

Read off and record the co-ordinates of the apexes of the resonant curves.

CO-ORDINATES OF RESONANCE FOR CONSTRAINED RFB				
%Qc	fr(h)		V	dpf at bus 5A
100%	fr(1)	3.833	19.018	Unity
	fr(2)	6.833	11.061	
80.774%	fr(1)	4.167	23.541	0.948
	fr(2)	7.000	12.110	
72.039%	fr(1)	4.333	24.772	0.900
	fr(2)	7.167	12.448	
25%	fr(1)	5.000	24.727	0.609
	fr(2)	11.000	9.358	

Table 6.4 Co-ordinates for resonance points of constrained RFB

DECISION THEORY PROCESS
FOR MAKING A
MITIGATION DECISION
ON
HARMONIC RESONANCE
IN
POWER SYSTEMS

GARY ATKINSON-HOPE

CHAPTER 6

The 5th harmonic curve has one “fr” [fr (1) = 5.0], while the 7th harmonic curve has two apexes within the RFB (equation 6.3), fr (1) = 4.167 fr (2) = 7.0, respectively.

Divide the RFB into two clusters of “fr” apexes, fr (1), excluding the 5th as is not a likely choice and fr (2) clusters.

Divide the fr (1) cluster into sub-regions and identify the relevant %Q_{C(an)} values, namely:

Sub-region A [fr(1) = 4.333 to 4.167]

$$\%Q_{C(an)(MIN)} \leq \%Q_{C(an)} < \%Q_{C(an)} [fr(hch) = 7^{th}] \quad (6.4)$$

Sub-region B [fr (1) = 4.167 to 3.833]

$$\%Q_{C(an)} [fr (hch) = 7^{th}] \leq \%Q_{C(an)} < \%Q_{C(an)(MAX)} \quad (6.5)$$

Identify the minimum number of “a_n” as decision-making is about choice and meaningful and feasible solutions.

Capacitors have tolerances, therefore within each sub-region select approximate midpoint (mpt) values, %Q_{C(mpt-A)} and %Q_{C(mpt-B)}, respectively.

Following the approach used in case 1 (chapter 5, equation 5.17), determine the midpoint capacitor sizes, namely:

$$Q_{C(mpt-A)} = 1323.4\text{kvars} (0.926\text{dpf}) (76.464\%)$$

$$Q_{C(mpt-B)} = 1565.5\text{kvars} (0.986 \text{ dpf})(90.386\%)$$

More “a_n” can be chosen if desired.

Superimpose onto the resonance curves of the four “a_n” shown in figure 6.5, the resonance curves for the two midpoint capacitor sizes (76.4% and 90.3%). See figure 6.6.

Six “a_n” are identified as possible choices for the capacitor size to be selected. As the effects of resonance can be damaging, the 5th (25% Q_C) and the 7th (80.774 %Q_C) are not chosen, leaving four possibilities, namely:

$$a_1 = \%Q_{C(MIN)} (72.04\%, 0.900 \text{ dpf})$$

$$a_2 = \%Q_{C(mpt-A)} (76.40\%, 0.926 \text{ dpf})$$

$$a_3 = \%Q_{C(mpt-B)} (90.30\%, 0.986 \text{ dpf})$$

$$a_4 = \%Q_{C(MAX)} (100.00\%, \text{unity dpf})$$

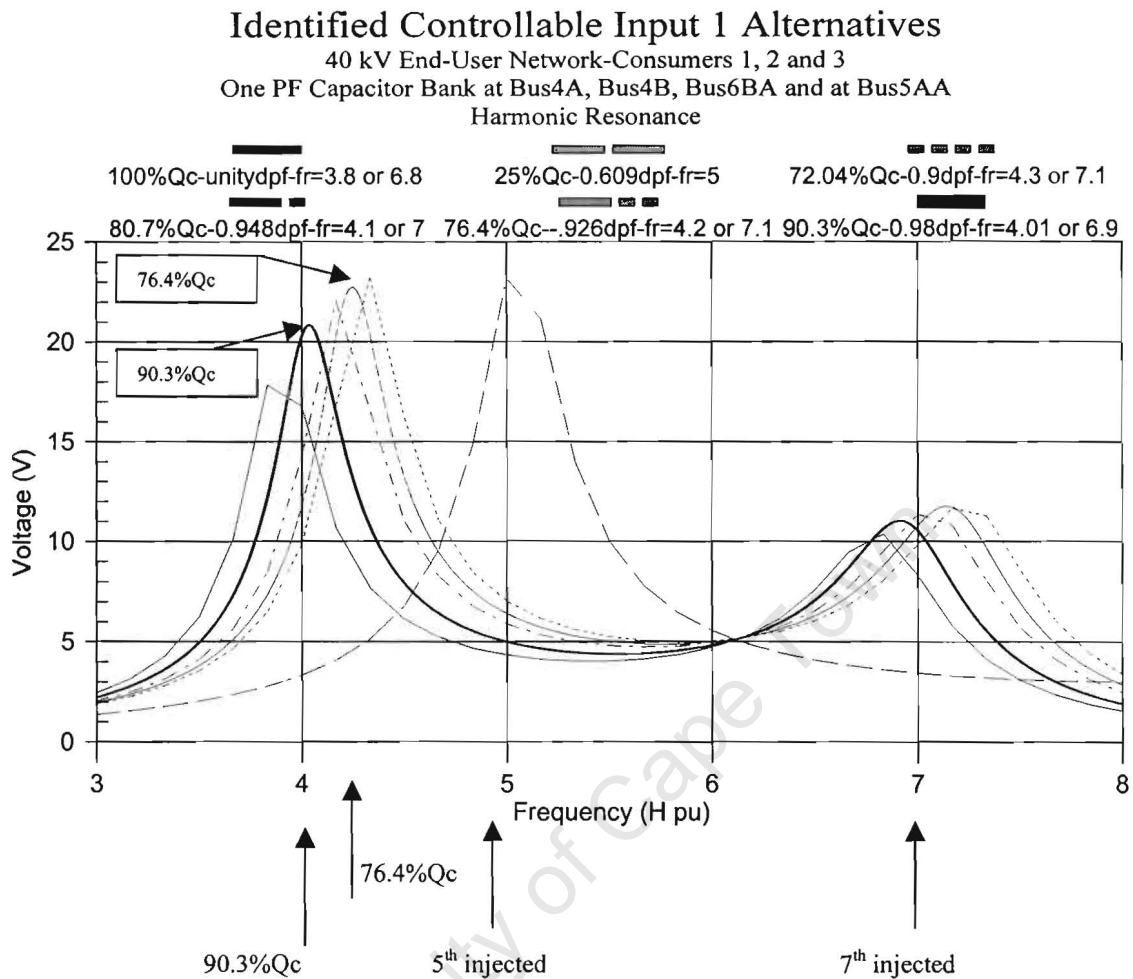


Figure 6.6 Identified controllable input 1, alternatives

In decision theory, a decision is best if limited to two choices.

It is traditional to improve a power factor to better than a dpf of 0.9 and also not to correct to unity. Therefore, a_1 and a_4 can be eliminated, leaving the two midpoint capacitor sizes, a_2 and a_4 as a constrained choice for controllable input 1 alternatives.

STEP 4:

Identify the controllable input 2, alternatives. As stated earlier, the CHRFB falls within the RFB, therefore the CHRFB is constrained to the 5th and 7th characteristic harmonics only, $hch = 6k \pm 1$, $k = 1$ only need to be considered.

Thus, only the 5th and 7th characteristics, injected by the sources need be considered in relation to the two controllable input 1, alternatives.

STEP 5:

Identify the 2-controllable input, alternatives for this case study. They are identified as follows:

2 – CONTROLLABLE INPUTS - ALTERNATIVES	
CONTROLLABLE INPUT 1	CONTROLLABLE INPUT 2
$a_2 [\%Q_{C(mpt-A)}]$	$a_{21}(5^{th})$
	$a_{22}(7^{th})$
$a_3 [\%Q_{C(mpt-B)}]$	$a_{31}(5^{th})$
	$a_{32}(7^{th})$

Table 6.5 The 2 – controllable inputs, alternatives

6.7 OBJECTIVE FUNCTION AND CALCULATION OF OUTCOMES (BLOCK D)

Develop the objective function taking into account constraints particular to case 2:

$$\%HRSI_{(sN)(anm)} = \frac{V_{C(sN)(anm)} [\%Q_{C(an)} (fr \neq hch)]}{V_{C(sN)(anm)} [\%Q_{C(an)} (fr = hch)]} \times 100\% \quad (6.6)$$

subject to: $hch = 6k \pm 1$, $k = 1$ only

$$anm = hch$$

$$\%Q_{C(MIN)} < \%Q_{C(an)(fr = hch)} < \%Q_{C(MAX)}$$

$$\%Q_{C(MIN)} = \%Q_{C(dpf = 0.9)}$$

$$\%Q_{C(MAX)} = \%Q_{C(dpf = 1)}$$

$$\%Q_{C(MIN)} \leq \%Q_{C(an)(fr \neq hch)} \leq \%Q_{C(max)}$$

$$\%Q_{C(a1)(fr \neq hch)} = \%Q_{C(mpt-A)}$$

$$\%Q_{C(a2)(fr \neq hch)} = \%Q_{C(mpt-B)}$$

$$\%Q_{C(mpt-A)} = \text{midpoint between } \%Q_{C(MIN)} \text{ and } \%Q_{C(7)(fr = 7)}$$

$$\%Q_{C(mpt-B)} = \text{midpoint between } \%Q_{C(7)(fr = 7)} \text{ and } \%Q_{C(MAX)}$$

$$s_N = s_1 (0.25FL), s_2(0.6FL) \text{ or } s_3 (1.0FL) \text{ only}$$

The two harmonic sources (drive 1 and 2) and the rest of the system (except end-user 1's linear load $S_{E(sN)}$) remain at full load values to represent the worst case scenario.

End-user 1's linear load ($S_{E(sN)}$) adjusted to s_1 , s_2 or s_3 only.

$\%Q_{C(mpt-A)}$ and $\%Q_{C(mpt-B)}$ values remain set at full load values when the states of nature are varied from s_1 to s_2 to s_3 .

Calculate the $\%HRSI_{(sN)(ann)}$ outcomes using the objective function and the following method.

- Develop “Harmonic Penetration Computer Models” with Cap5AA tuned to create 5th harmonic resonance at bus 5AA for the three states of nature, respectively. Do the same to create 7th harmonic resonance at bus 5AA for the three states of nature. From the results of each model, determine the voltages $V_{C(sN)} [fr = hch]$.
- Set Cap5AA value in the “Harmonic Penetration Computer Model” to a_2 and a_3 values, $\%Q_{C(mpt-A)}$ and $\%Q_{C(mpt-B)}$, respectively and calculate the $V_{C(sN)(ann)} [\%Q_{C(an)}(fr \neq hch)]$ voltage values for the three states of nature.
- The results and the $\% HRSI_{(sN)(ann)}$ calculations are given in Appendix 6, section 6.5. The outcomes are then represented in the decision table.

6.8 DEVELOP THE DECISION TABLE (BLOCK D)

SEVERITY OF HARMONIC RESONANCE DECISION TABLE - CASE 2				
DECISION ALTERNATIVES		STATES OF NATURE		
Controllable Inputs		Uncontrollable Inputs		
1	2	$s_1 = \text{minimum demand}$	$s_2 = \text{mean demand}$	$s_3 = \text{full load demand}$
a_2 $= \%Q_{C(mpt-A)}$	$a_{21} = 5^{th}$ $a_{22} = 7^{th}$	23.168% 135.838%	24.973% 129.372%	27.813% 121.943%
a_3 $= \%Q_{C(mpt-B)}$	$a_{31} = 5^{th}$ $a_{32} = 7^{th}$	18.364% 123.569%	19.660% 119.630%	21.716% 119.488%
PROBABILITIES		$P_{s1} = 0.3$	$P_{s2} = 0.6$	$P_{s3} = 0.1$

Table 6.6 Severity of harmonic resonance decision table – case 2

Using table 4.3 (chapter 4), eliminate a_{21} and a_{31} as they fall in the least severe category of harmonic resonance (objective is severe or very severe categories) to obtain the relevant decision table.

RELEVANT DECISION TABLE – CASE 2				
DECISION ALTERNATIVES		s ₁	s ₂	s ₃
a ₂	a ₂₂ = 7 th	135.838%	129.372%	121.943%
a ₃	a ₃₂ = 7 th	123.569%	119.630%	119.488%
PROBABILITIES		0.3	0.6	0.1

Table 6.7 Relevant decision table – case 2

On screening the relevant decision table it can be seen that all the outcomes fall in the very severe category. Also the decision-maker has preferences, therefore utility theory is needed to make decision 1.

6.9 UTILITY THEORY (BLOCK E)

Apply utility theory and the variable probability method to derive a utility table for the decision-maker. Then make a decision on the size of the capacitor bank as to which one is desirable/preferable in terms of severity of resonance.

The details for deriving the utility table and the utility function are given in Appendix 6, section 6.6.

Develop the utility table (risk averse).

UTILITY TABLE (RISK AVERSE) – CASE 2				
DECISION ALTERNATIVES		STATES OF NATURE		
		s ₁	s ₂	s ₃
a ₂	a ₂₂ = 7 th	0.0	0.600	0.96
a ₃	a ₃₂ = 7 th	0.8	0.985	1.00
PROBABILITIES		0.3	0.600	0.10

Table 6.8 Utility table (risk averse) – case 2

Calculate the expected utility values using equation 4.24 (chapter 4).

$$EU_{(a_2)(a_{22})} = 0.3(0) + 0.6(0.6) + 0.1(0.96) = 0.456$$

$$EU_{(a_3)(a_{32})} = 0.3(0.8) + 0.6(0.985) + 0.1(1) = 0.931 \text{ (highest)}$$

%Q_{C(a3)(a32)} is chosen. A capacitor size Q_C = 1565.5 kvars is preferred. The dpf for end-user 1, will be increased from 0.5 to 0.986.

Identify the severities from the decision table for the states of nature for the chosen capacitor size and record decision 1.

Decision 1

DECISION 1 IN THE PROCESS OF MAKING A MITIGATION DECISION					
Decision alternatives		States of nature			Expected utility Value
Controllable inputs		Uncontrollable inputs			
1	2	s ₁	s ₂	s ₃	EU
a _n =%Q _{C(an)} =1565.5 kvars (selected capacitor size)	a _{nm} =7 th (n th harmonic)	%HRSI _{(s1)(anm)} =123.569%	%HRSI _{(s2)(anm)} =119.630%	%HRSI _{(s3)(anm)} =119.488%	EU _{(an)(anm)} =0.931 (highest value)

Table 6.9 Mitigation decision 1 for case 2

6.10 MAKING A HARMONIC RESONANCE MITIGATION DECISION (BLOCK F)

As the severities are very high for the chosen capacitor size, it is necessary to check if mitigation is needed or not. That is, will the capacitor be damaged if installed into end-user 1's plant (Decision 2)?

- a. Carry out a harmonic penetration study for the three states of nature and calculate the $I_{C(RMS)}$ and V_{C1} values for the chosen capacitor once installed. The results are:

HARMONIC PENETRATION RESULTS		
s _N	I _{C(RMS)} (A)	V _{C1} (V)
s ₁	184.062	3650.65
s ₂	155.408	3573.64
s ₃	151.734	3489.14

Table 6.10 Harmonic penetration results

- b. Calculate the rated current and voltage at the fundamental frequency.

$$\begin{aligned}
 Q_{C(RATED)} &= 1565.54 \text{ kvars} \\
 U_{N(L-N)} &= 3464.1016 \text{ V} \\
 I_{L(RATED)} &= 150.644 \text{ A}
 \end{aligned}$$

- c. Calculate the mitigation index values. The calculations are shown in Appendix 6, section 6.7.

- d. Make decision 2.

Decision 2

DECISION 2 – MAKING A MITIGATION DECISION				
VARIABLES				
"s _N "	"anm"	%HRSI	%MI	%V _{C1}
s ₁	7 th	123.569%	93.986%	105.385%
s ₂		119.63%	92.858%	103.162%
s ₃		119.488%	91.637%	100.722%

Table 6.11 Mitigation decision 2 for case 2

The results in table 6.11 show that no mitigation is needed [$\%MI < 100\%$, $V_{C1} < 1.10U_N$]. After installation the capacitor will not be damaged and can be installed with confidence.

As a check, the IEEE standard loading indices for capacitors were applied (same approach followed as in Appendix 5, section 5.2 – case 1) and they confirm the finding that no mitigation is needed to prevent any damage to the capacitor.

6.11 OTHER MITIGATION CONCERNS

The focus of this research topic is the development of a decision theory process for making a mitigation decision on harmonic resonance for a pf correction capacitor in an end-user plant.

Nevertheless, when a capacitor is installed into a power system having an existing resonance, it will impact on the systems response to harmonic frequencies. Although this is not the focus of my research topic, some other mitigation concerns are briefly discussed.

6.11.1 IEEE 519 STANDARD

The question is should the IEEE 519 limits be considered [5], [16]?

The IEEE standard [16], provides that there are two parties in a power system concerned with harmonic flows. They are the utility and the end-user who injects harmonics into the system.

The IEEE 519 recommends limits to regulate harmonics in the power system. The utility is responsible for the VTHD% at the point of common coupling (PCC). The end-user who injects current harmonics is responsible in terms of the IEEE 519 for the magnitudes of harmonics injected. For PCC's below 69kV, the VTHD% limit is 5%. Table 10.3 of the IEEE standard provides the limits for individual harmonic current injections.

End-user's (e.g., end-user 1 in case 2) who do not inject current harmonics are therefore not responsible in terms of IEEE 519 for distortions. They are sinks for harmonics. However, they have a responsibility to work together with the utility and other end-users in a system.

It should be noted that a capacitor could be installed until the 130% and/or 110% limit for capacitors is reached. The IEEE 519 limits should be checked as the limits could already be exceeded at a smaller capacitor size. If exceeded, the utility and/or the end-users who inject harmonics will need to comply with the standard and not the end-user who has no drives but who installs a pf correction capacitor. Any changes introduced must not damage the pf capacitor to be installed. If the PCC IEEE 519 limit is within the 5%, the main concern for an end-user who has no drives is whether damage will occur to his pf capacitor due to severity of harmonic resonance. If so, then he is responsible for mitigation within his own plant.

The IEEE 519 should thus be used as a guideline to manage harmonics in a power system and is a recommended practice [2].

6.11.2 VIOLATION OF MITIGATION CRITERION (%MI)

If an end-user needs to employ mitigation in his plant, passive or active filters can be installed to diminish any severity of harmonic resonance problem encountered [5], [30].

Passive filters are comprised of a resistor, inductor and capacitor. Different types are available and can be shunt or series types. When introduced they alter the frequency response of the power system. Active filters are power electronic devices and have the advantage that they do not resonate with the system and therefore do not alter the frequency response of the system. Passive filters are widely used due to their simplicity and economical cost whereas active filters are more expensive. They are also more efficient devices for the reduction of harmonic levels. The disadvantage of commercially available active filters is that they are available only in discrete sizes and as such may not eliminate harmonics totally. [39]

The theory, design procedure and application of three types of passive filters are covered in the next chapter.

6.11.3 IEC 61000 – 2 – 4 STANDARD

The IEC standard 61000 – 2 – 4, class 2 applies to PCC's. It limits VTHD% to 8% as compared to 5% by the IEEE 519. Like, the IEEE 519, it also regulates individual harmonics injected by end-users, [5]. The IEC standard applies in South Africa.

6.11.4 VTHD% LIMIT AT PCC

In case 2, the VTHD% at the PCC (bus 3) is 11.5% before the installation of the new pf capacitor. This was probably due to the pre-existing 5th harmonic resonance in the system. After installation of the capacitor, the VTHD% decreased to 6.48%. The capacitor chosen by the decision theory process has thus decreased the VTHD% level at the PCC. The 6.48% value exceeds the IEEE 519 recommendation of 5% but is less than the IEC limit of 8%.

In terms of IEC, no action is needed. If working with the IEEE limit of 5% (only a recommendation), then the responsibility to improve the situation belongs to the utility or those end-users who inject harmonics. An active or passive filter could be employed. The utility or end-users that inject harmonics or both could be responsible for this improvement. The entire system, including the newly installed capacitor would need to be evaluated to determine the location of the mitigation device and who is responsible for its cost.

6.11.5 VOLTAGE LEVELS OF OTHER CAPACITORS

Besides considering the new capacitor installed in end-user 1's plant, the voltage levels of the other capacitors in the power system should be checked. There are capacitors at buses 4A, 4B and 6BA. Their levels were checked and the results are:

$$\begin{array}{ll} \text{BUS 4A} & 7021.91\text{V}(\omega_1) \quad U_{N(L-N)} = 6928.12\text{V} \\ \text{BUS 4B} & 7021.91\text{V}(\omega_1) \quad U_{N(L-N)} = 6928.12\text{V} \\ \text{BUS 6BA} & 230.65\text{V}(\omega_1) \quad U_{N(L-N)} = 230.90\text{V} \end{array}$$

Thus,

$$\begin{aligned} V_{\text{CAP}(4A)} &= V_{\text{CAP}(4B)} = 7021.91/6928.2 \times 100\% = 101.350\% \\ &= V_{\text{CAP}(6BA)} = 230.652/230.94 \times 100\% = 99.875\% \end{aligned}$$

All three capacitors are operating within $V_{C1} \leq 1.10U_N$.

The responsibility for voltage magnification issues at buses within the system rests with the utility, especially if there is more than one end-user in the system.

6.11.6 TRANSFORMER ENERGISATION

Magnetization currents (inrush) although rich in harmonics at normal operating voltages, are typically less than 1% of rated full load currents and only last for up to one second. Transformers are not as much of a concern as converters, which can produce harmonic currents of 20% of their rating.

When energizing a transformer, the inrush current contains even (e.g., 2nd and 4th harmonics) as well as odd harmonics lasting up to one second. The fundamental component is dominant and the 2nd and 4th harmonic voltages have magnitudes of approximately 10% and 5% of the fundamental frequency voltage. Voltage harmonics resulting from inrush currents depend on the network configuration and operating conditions [40].

These voltages (e.g., 4th) could be magnified if resonance is excited when the transformer is simultaneously energized with power factor correction capacitors.

The resonance near the 4th shown in figure 6.6 could be an issue during transformer energization if the capacitor bank in end-users 1 plant is in operation. This could result in an over voltage which could exceed acceptable limits. This dynamic over voltage (e.g., 4th) problem can be eliminated simply by not energizing the consumer transformer and capacitor together [11]. Thus transformers and capacitors should not be energized together.

6.11.7 BACKGROUND HARMONICS

Parallel resonance within a given consumer system involves internally generated harmonics (e.g. drives), giving rise to resonance between the local capacitance and the

predominantly supply inductance which are in parallel. The harmonic currents flow from the internal harmonic source through parallel paths and split in accordance with the impedance ratios. Series resonance typically refers to resonance between the external harmonic source (background harmonics) beyond the point of common coupling and capacitors in a consumer system. This implies that the external source is a harmonic voltage and that the resonance circuit involves a portion of the supply circuit, plus the consumers supply transformer in series with his capacitor. External supply voltage harmonics having a magnitude of less than or equal to 1% are seldom a problem.

If background harmonics from beyond the PCC are less than 1%, then they are usually ignored and the harmonics arriving at end-user 1 originate from the internal harmonic source (drives in end-user 3).

Any series combination of capacitor banks, lines and transformers can give rise to series resonance and can result in high levels of voltage (depending on damping) within the circuit and should be avoided. Series resonance is one of the less common conditions known to cause harmonic problems [16]. If there is a problem it is the utilities responsibility to provide the solution [12].

6.12 SUMMARY

It is found that two controllable inputs, capacitor size and characteristic harmonics are required per decision alternative when investigating the severity of harmonic resonance.

The developed Decision Table is found to be an effective quantitative model to structure and represent the decision problem involving the severity of harmonic resonance.

The %HRSI and %MI Indices are shown to be effective for evaluating the severity of harmonic resonance and for making a mitigation decision, respectively.

The results show that the developed three-stage decision theory process, is a rational methodology for conceptualising, analysing and solving a decision problem involving harmonic resonance mitigation for an end-users plant.

For the case study investigated it is found that no mitigation device needs to be installed in the end-user's plant for the chosen capacitor size, despite that the severities for the three states of nature are very severe. The IEEE standard loading indices for capacitors should always be used to check and confirm the mitigation finding. In this case, the IEEE indices confirm the finding that no mitigation is needed.

Using a real system for the application of the developed decision theory process, it is revealed that it is not always necessary to mitigate harmonic resonance for pf capacitor applications even if very severe harmonic resonance is found in the system.

No model can be an exact replica of a real system. However, the computer models developed for case 2 gave similar results to the field measurements for the worst case

CHAPTER 6

resonance scenario. A good model should be a realistic approximation. The decision model developed for case 2 is found to be realistic and its application as part of the decision theory process has led to a feasible and meaningful solution and is therefore used as a benchmark for case 3.

Other mitigation concerns not directly relevant to the topic researched have been briefly addressed for completeness.

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CHAPTER 7

CASE STUDY 3

INVESTIGATION OF THE EFFECT OF DAMPING ON THE SEVERITY OF RESONANCE AND ON THE MITIGATION DECISION IN A POWER SYSTEM HAVING MULTIPLE RESONANT POINTS

In this chapter a third case study is conducted. It is based on the same system as case 2, except that damping is decreased by reducing the resistance value of the X/R ratio of the transformer directly supplying end-user 1. The purpose is to investigate the effect that damping has on the %HRSI and %MI values and how the change influences the mitigation decision. The developed decision theory process is applied. Scan curves are generated for the decision alternatives to show how the decrease in damping increases the apex values of the resonance points. The %HRSI outcomes are calculated and the decision and utility tables are formulated and Decision 1 is made. The %MI values are calculated and Decision 2 identifies that mitigation is needed. The theory and design procedures for three different types of passive filters are reviewed in Appendix 7. Three possible mitigation solutions are investigated and filters are designed for them. Scan and penetration results are generated to show the effect the filters have on mitigation. A solution is recommended.

7.1 BACKGROUND

The same power system used in case 2 is investigated in this case study (figure 6.1). The computer scan and penetration models for the system used for case 2 are given in appendix 6. The “R” and “X” values of transformer (TI) are $0.9885\ \Omega$ and $12.0\ \Omega$ (X/R ratio = 12.139), respectively. In this case, a change in damping, that is, how a decrease in the “R” value of the X/R ratio of “TI” supplying end-user 1 (Load 5A) affects the $\%HRSI_{(sN)(anm)}$ and $\%MI_{\%HRSI_{(sN)(anm)}}$ values and influences the mitigation decision. Possible solutions to the mitigation problem are discussed.

7.2 DECISION SCENARIO

End-user 1 wants to improve his low dpf of 0.5 by installing a power factor correction capacitor bank. End-user 1 has been advised by the utility that the transformer supplying his plant is to be replaced. The new transformer will have all the same parameters as in case 2, except its “R” value will be decreased from $0.9885\ \Omega$ to $0.5\ \Omega$ (X/R = 24).

He is aware of other capacitors in the system and that end-user 3 has two 6-pulse drives.

He is concerned that his new capacitor could be damaged by resonance and that there may be a need for mitigation. He is aware that the level of power drawn by his plant impacts on the severity of harmonic resonance. The end-user is further aware that he needs a decision-maker to decide on the size of capacitor to be installed and to make a decision on mitigation.

7.3 DEFINE THE PROBLEM (BLOCK A)

The problem is whether the change in damping causes the resonance to be severe enough to damage the capacitor to be installed when the plant is operating at minimum, mean or full load operating conditions.

7.4 OBJECTIVES (BLOCK A)

Decision 1

Determine the $\%HRSI_{(sN)(ann)}$ outcomes for the three states of nature and make a decision as to what size of capacitor should be chosen, taking into account the preferences and desires of the decision-maker.

Decision 2

If severe levels of severity of harmonic resonance are found, make a mitigation decision and if necessary, suggest possible solutions.

7.5 STATES OF NATURE (BLOCK B)

The same states of nature as identified in case 2 apply to this case.

7.6 DECISION ALTERNATIVES (BLOCK C)

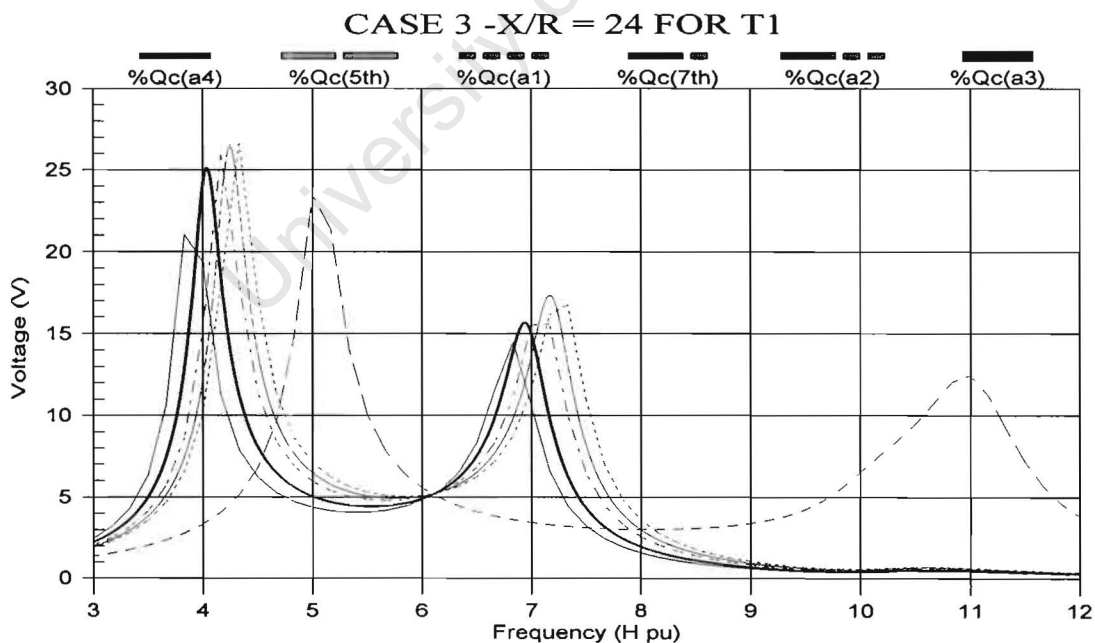


Figure 7.1 Resonance curves $X/R = 24$ - case 3

CHAPTER 7

As there is no change to any “X” values in the system, the resonance points for the controllable input 1, alternatives are the same as for case 2 (see figure 7.1), except all apex values increase due to the decrease in damping.

For the same reasons given in case 2, the 5th, 7th, a_1 and a_4 inputs can be eliminated, leaving a_2 and a_3 only. The objective function is also the same as for case 2.

7.7 RELEVANT DECISION TABLE (BLOCK D)

The relevant decision table for case 3 is:

RELEVANT DECISION TABLE – CASE 3				
DECISION ALTERNATIVES		STATES OF NATURE		
1	2	s_1	s_2	s_3
a_2	$a_{22} = 7^{\text{th}}$	90.445%	89.046%	87.765%
a_3	$a_{32} = 7^{\text{th}}$	87.342%	91.480%	96.579%
PROBABILITIES		0.3	0.6	0.1

Table 7.1 Relevant Decision Table – case 3

The outcomes all fall in the severe category and are all very similar in value, making decision 1 not obvious.

7.8 UTILITY TABLE (BLOCK E)

The utility table for case 3 is:

UTILITY TABLE – CASE 3				
DECISION ALTERNATIVES		STATES OF NATURE		
CONTROLLABLE INPUTS		UNCONTROLLABLE INPUTS		
1	2	$s_1 = 0.25$ full load demand	$s_2 = 0.6$ full load demand	$s_3 = 0.1$ full load demand
a_2 = % $Q_{C(\text{mpt-A})}$	$a_{22} = 7^{\text{th}}$	0.84	0.91	0.95
a_3 = % $Q_{C(\text{mpt-B})}$	$a_{32} = 7^{\text{th}}$	1.0	0.8	0.0
PROBABILITIES		0.3	0.6	0.1

Table 7.2 Utility Table - case 3

If needed, the utility function can be derived by using the utility and the %HRSI values. Calculate the $EU_{(an)(anm)}$ values for each capacitor size.

$$EU_{(a_2)(7^{\text{th}})} = (0.3)(0.84) + (0.6)(0.91) + (0.1)(0.95) = 0.893 \text{ (highest)}$$

$$EU_{(a_3)(7^{\text{th}})} = (0.3)(1.0) + (0.6)(0.8) + (0.1)(0.0) = 0.780$$

The capacitor size chosen is therefore $a_2 = 1324.4\text{kvars}$. In case 2, $a_3 = 1565.5\text{kvars}$ was chosen. For a_2 the severities are: $90.445\%(s_1)$, $89.046\%(s_2)$ and $87.765\%(s_3)$, respectively.

Decision 1

DECISION 1 IN THE PROCESS OF MAKING A MITIGATION DECISION					
Decision alternatives		States of nature			Expected utility Value
Controllable inputs		Uncontrollable inputs			
1	2	s ₁	s ₂	s ₃	EU
a _n =%Q _{C(an)} =1324.4 kvars (selected capacitor size)	a _{nm} = 7 th (m th harmonic)	%HRSI _{(s₁)(anm)} = 90.445%	%HRSI _{(s₂)(anm)} = 89.046%	%HRSI _{(s₃)(anm)} = 87.765%	EU _{(an)(anm)} = 0.893 (highest value)

Table 7.3 Mitigation decision 1 - case 3

7.9 DECISION 2 (BLOCK F)

Calculate the rated current and voltage for the chosen capacitor at the fundamental frequency:

$$Q_C = 1324.4\text{kvars}, V_{L-N} = 3464.1016\text{V}, I_{\text{RATED}} = 150.6445\text{A}$$

Conduct a harmonic penetration study for the three states of nature to calculate $I_{C(RMS)}$ and V_{C1} values. Calculate %MI indices with the capacitor size set to 1324.4kvars (see Appendix 7, section 7.1). Decision 2 is then made.

Decision 2

DECISION 2 – MAKING A MITIGATION DECISION				
VARIABLES				
" s_N "	"anm"	%HRSI	%MI	$\%V_{C1}$
s_1	7^{th}	90.445%	104.86%	104.56%
s_2		89.046%	102.41%	102.41%
s_3		87.765%	99.62%	100.046%

Table 7.4 Mitigation decision 2 - case 3

Decision 2 shows that mitigation is needed as %MI for s_1 and s_2 both exceed 100%. If the capacitor is installed it could be damaged.

To confirm the finding that mitigation is needed, check the results against the IEEE standard loading indices for capacitors. The harmonic penetration results are as follows:

$$s_1: V_{C(RMS)} = 3651.97\text{V}, I_{C(RMS)} = 173.729\text{A}, 634.453\text{kvars}$$

$$s_2: V_{C(RMS)} = 3576.75\text{V}, I_{C(RMS)} = 169.574\text{A}, 606.523\text{kvars}$$

$$s_3: V_{C(RMS)} = 3494.16\text{V}, I_{C(RMS)} = 165.044\text{A}, 576.690\text{kvars}$$

$$\text{Capacitor/phase rating: } 1324.4 \div 3 = 441.466\text{kvars}$$

The IEEE standard loading indices for capacitors are summarized in Appendix 1. Only the IEEE kvar index ($\leq 135\%$) check is reported here, namely:

$$\text{IEEE kvar}_{(s1)} = 634.453/441.446 = 143.715\% (\geq 135\%) \text{ mitigation needed}$$

$$\text{IEEE kvar}_{(s2)} = 606.523/441.466 = 137.380\% (\geq 135\%) \text{ mitigation needed}$$

$$\text{IEEE kvar}_{(s3)} = 576.69/441.466 = 130.630\% (\leq 135\%) \text{ no mitigation needed}$$

The calculated IEEE kvar index values confirm the finding that mitigation is needed.

7.10 VTHD % AT PCC

In figure 6.1, there are three end-users and they are connected to bus 3(PCC). Using the penetration results, obtain the VTHD% values at the PCC.

$$s_1: \text{VTHD}\% = 6.34\%$$

$$s_2: \text{VTHD}\% = 5.99\%$$

$$s_3: \text{VTHD}\% = 5.63\%$$

The IEEE 519 limit (5%) is exceeded but results are within the IEC 61000-2-4 limit (8%).

7.11 MITIGATION OF HARMONICS

Before discussing some possible mitigation solutions for case 3, the basic principles of mitigation of harmonics are reviewed.

There are many devices available to mitigate harmonics in power systems. They can be a simple combination of a capacitor bank and a reactor or as complex as an active filter. The simple solution should always be explored before considering a more complex device [30]. The basic principles of three passive filters (series tuned filter, notch filter and 2nd order damped filter) are reviewed in Appendix 7. The latter two filters are applied to demonstrate possible mitigation solutions for case 3.

7.12 MITIGATION SOLUTIONS FOR CASE 3

As stated in chapter 6, section 6.11, mitigation solutions is not the focus of this research topic. For completeness, three possible solutions are investigated.

When considering a mitigation solution the first approach should be to mitigate harmonics injected by a harmonic source. The mitigation device should be connected to the same bus as the harmonic source. The device is usually a filter and can be a passive or an active filter. If a passive filter is used then it is usual to use the pf capacitor connected

CHAPTER 7

to the same bus as the harmonic source as part of the filter thereby effecting a large cost saving.

In case 3, however, there are no pf capacitors at the same bus as the harmonic sources (end-user 3), therefore the above option is not considered and an alternative solution is sought. The solution is therefore to make use of existing capacitors in the system if passive filters are to be employed. The options are the capacitors (CAP 4A and 4B) at the PCC and at bus 6B (part of end-user 3) as well as the new pf capacitor to be installed at bus 5AA.

The following factors also need consideration:

a. In terms of IEEE 519, the VTHD% must be less than 5% at the PCC, and either the utility or the end-user who injects harmonics (end-user 3) should take action. If the utility takes responsibility then the capacitors at the PCC should be employed. If end-user 3 takes responsibility then the capacitor at bus 6B should be employed.

b. In terms of IEC 61000-2-4 standard, the VTHD% at the PCC must be less than 8%. As stated in “other mitigation concerns” (chapter 6, section 6.11):

“ If the VTHD% at the PCC is within the limit prescribed by the standard, then the utility and the end-user who injects harmonics are not responsible to do mitigation. The onus is then on the end-user himself for taking mitigation action to prevent his capacitor from being damaged. He has however an obligation to advise the utility of his plans so that they can check to see if the VTHD% at the PCC stays within the limit.”

There are three possible actions:

(1) Utility responsibility, as the system has three end-users and it has capacitors installed at the PCC (IEEE 519 standard).

(2) End-user 3 responsibility, that is, use the capacitor at bus 6B (IEEE 519 standard).

(3) End-user 1 responsibility by using the new capacitor at bus 5AA on the basis that the IEC61000-2-4 standard applies to the system.

The objectives for all three actions is to decrease the %MI values for the states of nature below 100% for the capacitor in end-user 1 and to decrease the VTHD% below 5% for the two IEEE 519 options and for solution 3 to maintain it below 8% for the IEC option with the possibility of bringing it within the IEEE 519 limit.

7.13 MITIGATION SOLUTION 1

Design a 2nd – order harmonic filter using the two capacitors at the PCC. The fundamental frequency is 60Hz (see Harmonic filter theory – Appendix 7, section 7.2.3).

Step 1. Connect the two capacitors in parallel,

$$C_{TOT} = 4\text{Mvars (C = 663.145pf)}$$

Step 2. Let $h_n \geq 10.7$

$$\text{Step 3. } X_C = \frac{12^2}{4} = 36\Omega$$

$$\text{Step 4. } X_L = \frac{36}{10.7^2} = 0.31443\Omega (L = 0.83405\text{mH})$$

$$\text{Step 5. } X_n = \sqrt{(0.31443)(36)} = 3.36444 \Omega$$

Step 6. Applying the recommendation that a Q of 1 or 2 should be used, select Q = 1, then,

$$R = 3.36444 \times 1 = 3.36444\Omega$$

Step 7. The parameters are therefore:

$$R = 3.36444\Omega, L = 0.83405\text{mH}(0.31443\Omega), C = 663.145\text{pf}(4\text{Mvars})$$

They are to be used to model the filter for the software investigation.

Step 8. The configuration for the filter is thus a resistor in parallel with the reactor and then both in series with the capacitor bank.

The computer model with the 2nd – order damped filter included at the PCC is given in Appendix 7, section 7.3. Conduct scan studies for the installed filter and without the filter. Superimpose the two curves on a common axis to see the filtering and damping actions and the effects at the PCC and at bus 5AA.

The results are given below in figures 7.2 (PCC) and 7.3 (bus 5AA), respectively:

SCAN RESULTS AT PCC
40 kV End-User Network-Consumers 1, 2 and 3
Capacitor Bank in filter, Bus6BA and at Bus5AA
Harmonic Resonance - MPT-A

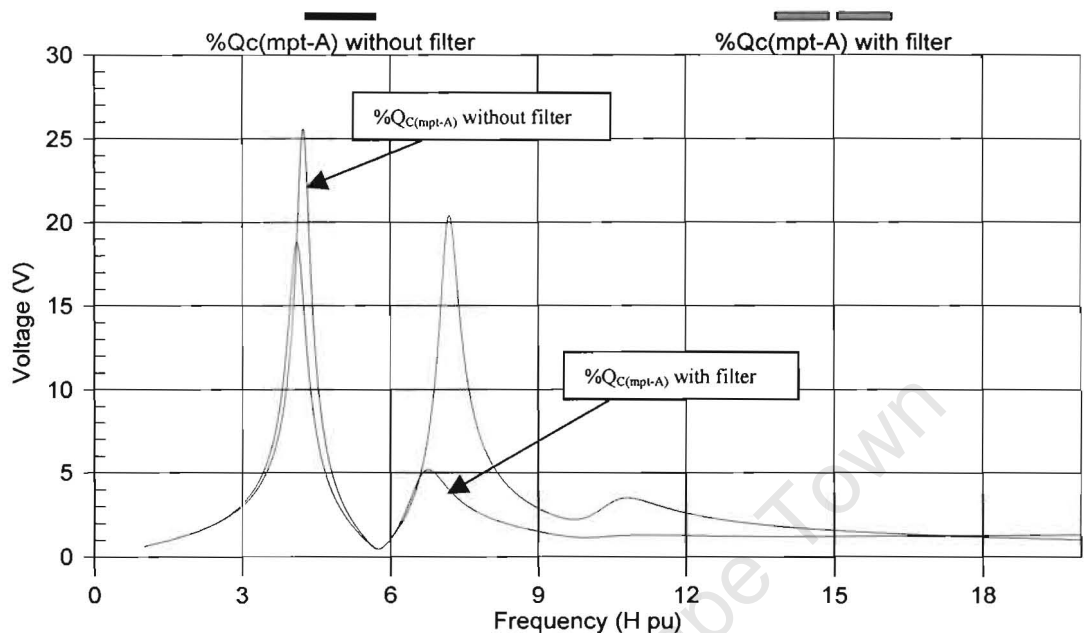


Figure 7.2 Scan results at PCC

SCAN RESULTS AT BUS5AA
40 kV End-User Network-Consumers 1, 2 and 3
Capacitor Bank in filter, Bus6BA and at Bus5AA
Harmonic Resonance - MPT-A

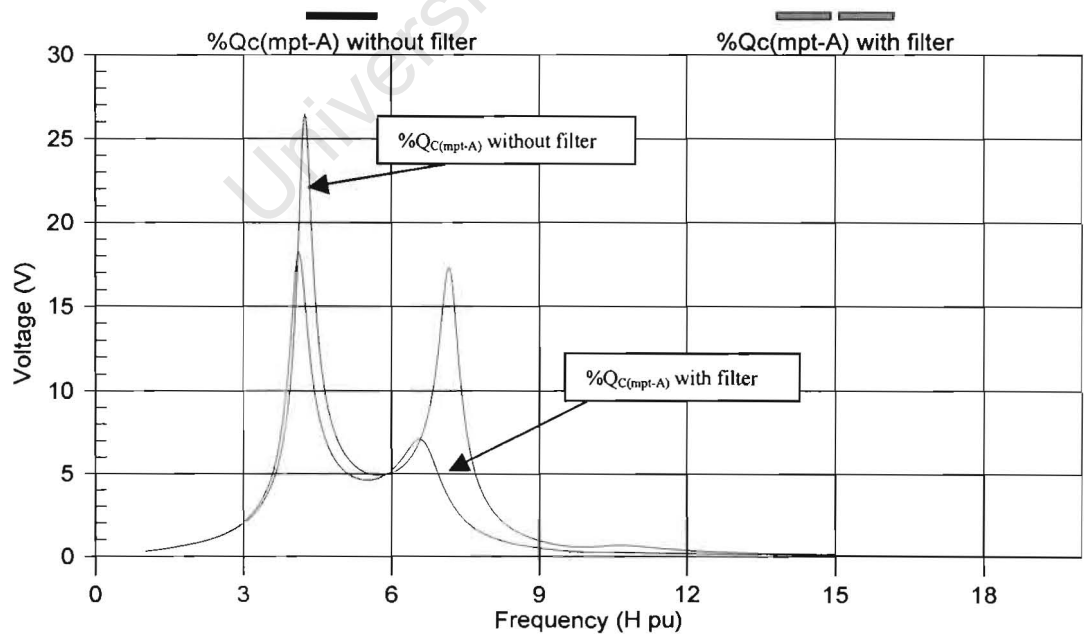


Figure 7.3 Scan results at bus 5AA

It can be seen that at both the PCC (figure 7.2) and at bus 5AA (figure 7.3) where the new capacitor is installed, the apex values are decreased when the filter is in operation.

At the PCC without the filter, the apex values are 25.364V and 20.349V, respectively.

With the filter, the resonance points are shifted, the impedance ($Z \propto V$) is more flat from the 10.7th and the apexes are decreased to 18.812V and 5.109V, respectively.

At bus 5AA without the filter, the apex voltage values are 26.344V and 17.245V, respectively. With the filter, the apex values decreased to 18.185V and 7.173V, respectively.

The effect of the 2nd – order damped filter (10.7th tuning) can be seen. The values below the 10.7th point increase offering higher impedances to low order harmonics (5th and 7th) and a low impedances to those >10.7th.

After conducting harmonic penetration studies with the filter included in the computer models for the three states of nature the following VTHD% (PCC) and %MI (bus 5AA) results were obtained (see Appendix 7, section 7.4):

2 ND ORDER HARMONIC FILTER RESULTS-MITIGATION SOLUTION 1							
S _N	VTHD%	I _{C(RMS)} (A)	1.3I _{1(RATED)} (A)	%MI	V _{C1} (V)	V _{C1(RATED)} (V)	%V _{C1}
s ₁	2.20	141.72	165.60	85.54	3623.35	3464.10	104.59
s ₂	2.29	139.45	165.60	84.17	3548.72	3464.10	102.44
s ₃	2.39	137.03	165.60	82.71	3466.71	3464.10	100.67

Table 7.5 2nd Order harmonic filter results-mitigation solution 1

The %MI values are all less than 100%, showing that mitigation is successful. The % V_{C1} values are also all less than 110% U_N. The VTHD% at the PCC is reduced to below the 5% limit required by IEEE 519.

As the filter is installed at the PCC, the utility will be responsible for the cost and upgrade and this solution will benefit all three end-users connected to the PCC.

7.14 MITIGATION SOLUTION 2

Design a notch filter using the single capacitor bank at end-user 3's, bus 6B. The fundamental frequency is 60Hz (see Appendix 7, section 7.2.2).

Q_C = 300kVars, kV = 0.4, hch = 6k±1, k = 1,2 and 3 only.

Step 1.
$$X_C = \frac{0.4^2}{0.3} = 0.53333\Omega$$

Step 2. $C_f = 4973.623\mu\text{F}$

Step 3. Let $x = 0.06(6\%)$

$$H_{\text{notch}} = 300 - [(0.06)(300)] = 282\text{Hz (4.7}^{\text{th}})$$

Step 4. $L_f = \frac{1}{(2\pi 282)^2 4973.623 \times 10^{-6}} = 0.06404\text{mH}$

Step 5. $X_R = 2\pi(60)(0.06404 \times 10^{-3}) = 0.024143\Omega$, or

$$X_R = \frac{0.53333}{4.7^2} = 0.024143\Omega$$

Step 6. Let, $Q = 30$, then

$$R_R = \frac{(282)(0.024143)}{30} = 0.22694\Omega$$

The notch filter parameters are:

$$R_R = 0.22694\Omega, L_f = 0.06404\text{mH}(X_R = 0.024143\Omega), C_f = 4973.623\mu\text{F}(X_c = 0.5333\Omega).$$

Step 7. The configuration is a resistor, inductor and capacitor in series.

The computer model with the notch filter included at bus 6B is given in Appendix 8.

The scan results for bus 5AA are given in figure 7.4:

SCAN RESULTS NOTCH FILTER

40 kV End-User Network-Consumers 1, 2 and 3
One PF Capacitor Bank at Bus4A, Bus4B and at Bus5AA
Harmonic Resonance - MPT-A Notch filter at bus6B

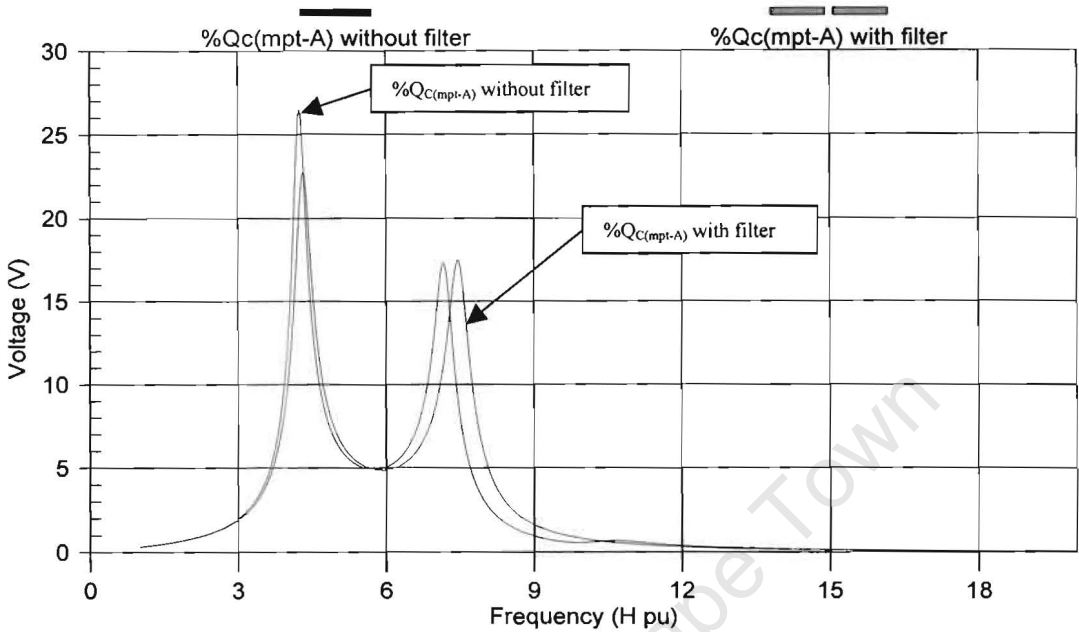


Figure 7.4 Scan results notch filter at bus 6B

Without the filter there is a resonance near the 7th harmonic. At the 7th characteristic harmonic the voltage level is 13.655V. With the filter the resonance frequency points have shifted to the right and the one nearest the 7th has shifted further away and the voltage at the 7th harmonic is decreased to 8.450V.

The notch filter at bus 6B has caused the 7th harmonic voltage at bus 5AA to decrease.

After conducting harmonic penetration studies with the filter included in the computer models for the three states of nature, the following VTHD% (PCC) and %MI (bus 5AA) results were obtained (see Appendix 8, section 8.2):

NOTCH FILTER RESULTS-MITIGATION SOLUTION 2							
S _N	VTHD%	I _{C(RMS)} (A)	1.3I _{1(RATED)} (A)	%MI	V _{C1} (V)	V _{C1(RATED)} (V)	%V _{C1}
s ₁	4.08	154.38	165.67	93.19	3620.06	3464.10	104.51
s ₂	4.01	152.05	165.67	91.78	3546.06	3464.10	102.36
s ₃	3.96	149.63	165.67	90.31	3464.12	3464.10	100.00

Table 7.6 Notch filter results-mitigation solution 2

CHAPTER 7

From the harmonic penetration results the capacitor voltages at bus 6B (end-user 3) are:

$$(s_1) V_{C1} = 222.311V \quad V_{C1(RATED)} = 230.94V \\ \%V_{C1} = 96.263\% \quad (V_{C1} \leq 1.10U_N)$$

$$(s_2) V_{C1} = 221.066V \quad V_{C1(RATED)} = 230.94V \\ \%V_{C1} = 95.774\% \quad (V_{C1} \leq 1.10U_N)$$

$$(s_3) V_{C1} = 219.693V \quad V_{C1(RATED)} = 230.94V \\ \%V_{C1} = 95.129\% \quad (V_{C1} \leq 1.10U_N)$$

The %MI values are all less than 100% showing that mitigation is successful for end-user 1. The % V_{C1} values at both end-user 1 and 3 are also less than 110% U_N , thus the capacitors are not at risk. The VTHD% at the PCC is reduced to below the 5% limit required by the IEEE 519. The capacitors at the PCC are therefore also not at risk.

As the filter is installed in end-user 3's plant, he will be responsible for the cost and upgrade. He is one of the two parties identified in the IEEE 519 as a responsible person, the utility and the end-user who injects harmonics.

As the VTHD% is reduced below 5% all three end-users will benefit from this mitigation solution.

7.15 MITIGATION SOLUTION 3

Design a notch filter using the single capacitor bank at end-user 1's bus 5AA. The fundamental frequency is 60Hz.

$$Q_C = 1.3244\text{Mvars, kV} = 6, hch = 6k \pm 1, k = 1, 2 \text{ and } 3 \text{ only.}$$

$$\text{Step 1.} \quad X_C = \frac{6^2}{1.3244} = 27.1821\Omega$$

$$\text{Step 2.} \quad C_f = 97.5855\mu\text{F}$$

$$\text{Step 3.} \quad \text{Let } x = 0.06(6\%)$$

$$H_{\text{notch}} = 300 - [(0.06)(300)] = 282\text{Hz (4.7}^{\text{th}})$$

$$\text{Step 4.} \quad L_f = \frac{1}{(2\pi 282)^2 97.5855 \times 10^{-6}} = 3.26404\text{mH}$$

$$\text{Step 5.} \quad X_R = 2\pi(60)(3.26404 \times 10^{-3}) = 1.2305\Omega$$

$$\text{or } X_R = \frac{27.1821}{4.7^2} = 1.2305\Omega$$

Step 6. Let, $Q = 150$, to keep I^2R losses as low as possible and to bring PCC VTHD% within $<5\%$, then,

$$R_R = \frac{(282)(1.2305)}{150} = 2.3133\Omega$$

The notch filter parameters are:

$$R_R = 2.3133\Omega, L_f = 3.26404\text{mH}(X_R = 1.2305\Omega), C_f = 97.5855\mu\text{F}(X_c = 27.1821\Omega).$$

The computer model is upgraded to include the 4.7th notch filter at bus 5A. The scan results are given in figure 7.5:

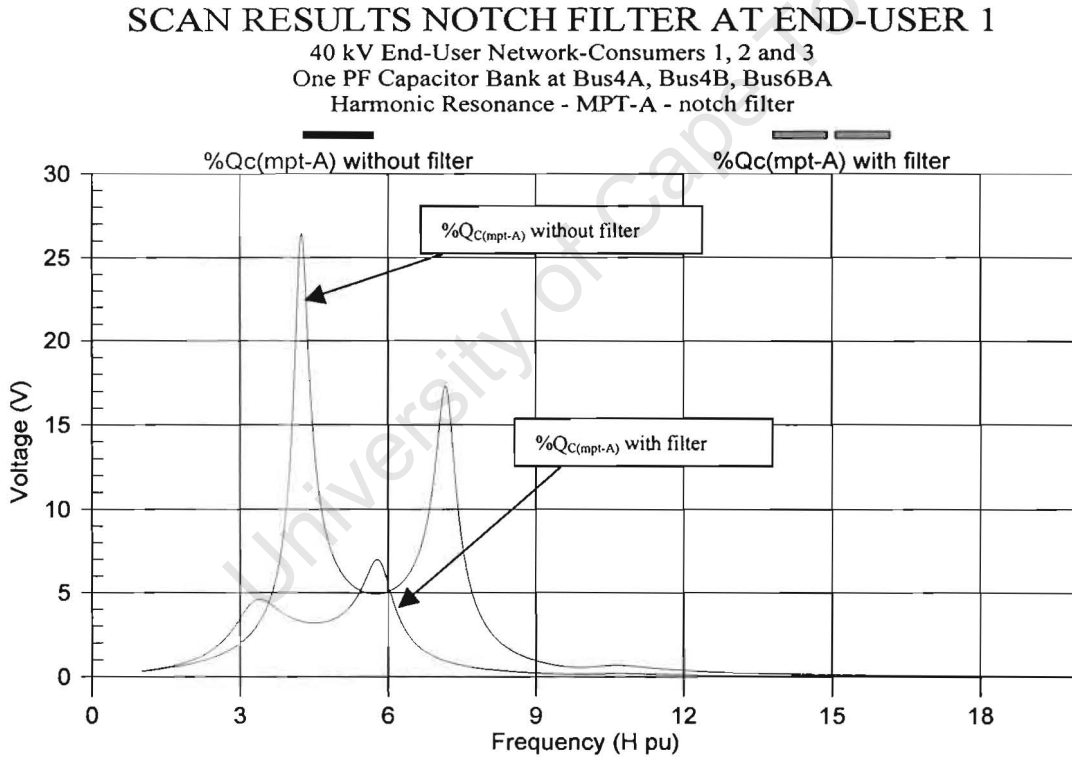


Figure 7.5 Scan results notch filter bus5AA

The resonance points are shifted to the left and the apex values are reduced to 4.620V and 6.811V, respectively. At the 7th characteristic harmonic the voltage is reduced even more.

After conducting harmonic penetration studies with the filter included in the computer models for the three states of nature the following VTHD% (PCC) and %MI (bus 5AA) results were obtained (see Appendix 8, section 8.3):

NOTCH FILTER RESULTS-MITIGATION SOLUTION 3							
S _N	VTHD%	I _{C(RMS)} (A)	1.3I _{1(RATED)} (A)	%MI	V _{C1} (V)	V _{C1(RATED)} (V)	%V _{C1}
s ₁	4.96	142.01	165.67	85.72	3784.54	3464.10	109.24
s ₂	4.97	139.13	165.67	83.98	3706.33	3464.10	106.99
s ₃	4.97	135.97	165.67	82.07	3620.40	3464.10	104.51

Table 7.7 Notch filter results-mitigation results 3

From the harmonic penetration results the capacitor voltages at bus 6B (end-user 3) are:

$$(s_1) V_{C1} = 233.017V \quad V_{C1(RATED)} = 230.94V \\ \%V_{C1} = 100.899\% \quad (V_{C1} \leq 1.10U_N)$$

$$(s_2) V_{C1} = 231.704V \quad V_{C1(RATED)} = 230.94V \\ \%V_{C1} = 100.33\% \quad (V_{C1} \leq 1.10U_N)$$

$$(s_3) V_{C1} = 230.258V \quad V_{C1(RATED)} = 230.94V \\ \%V_{C1} = 99.7\% \quad (V_{C1} \leq 1.10U_N)$$

The %MI values are all less than 100% showing that mitigation is successful for end-user 1. The %V_{C1} values at both end-user 1 and 3 are also less than 110% U_N, thus the capacitors are not at risk. The VTHD% at the PCC is reduced to below the 5% limit required by the IEEE 519. The capacitors at the PCC are therefore also not at risk.

End-user 3 is not one of the parties identified by IEEE 519 responsible for harmonic distortion. In terms of IEC 61000-2-4, the VTHD% without any filter was less than the required 8%. In terms of IEC standard end-user 1 would be responsible for the cost and upgrade, but would have an obligation to advise the utility of his plans. Nonetheless, the notch filter reduces the VTHD% at the PCC below the mandatory 5% and is therefore also a possible solution.

7.16 RECOMMENDED MITIGATION SOLUTION

As the IEEE 519 is a stricter standard than the IEC standard, it is recommended that it be applied to the system. This eliminates mitigation solution 3 from the options.

As there are three end-users connected to the PCC, mitigation solution 1 is recommended as the solution to be implemented. The utility has existing capacitors at the PCC and in terms of the IEEE 519 is responsible for the VTHD% at the PCC. After the filter has been installed at the PCC, end-user 1 can install and operate his new capacitor.

7.17 SUMMARY

The developed decision theory process is applied to a power system in which the damping has been decreased and it is found that mitigation is needed. The VTHD% at the PCC is found to exceed the IEEE 519 limit but to be within the IEC 61000-2-4 standard.

The theory and procedure for designing three types of passive filters are reviewed in Appendix 7.

Three possible mitigation solutions are investigated. Their purpose is not only to ensure that the new pf capacitor to be installed in end-user 1's plant will not suffer damage but also to bring the VTHD% at the PCC within the IEEE 519 limit.

A filter is designed for each solution. A computer model is developed for each filter and included in the main computer model used for investigating the decision problem.

It is found that all three mitigation solutions will prevent the new pf capacitor in end-user 1's plant from becoming damaged. In all three solutions, the VTHD% is also reduced to below the 5% limit as prescribed by the IEEE 519. Also, all three solutions satisfy the IEC 61000-2-4 standard. If the IEC 61000-2-4 standard applies to the system, then mitigation solution 3 is found to be the most relevant solution.

As the IEEE 519 standard is stricter than the IEC 61000-2-4, the former should rather be applied. Therefore, for case 3 it is found that mitigation solution 1 is the most suitable and should be implemented as the solution.

CHAPTER 8

CLOSURE

Contributions, conclusions, recommendations and future work are identified.

8.1 CONTRIBUTIONS

Decision analysis traditionally applies to business and not to electrical engineering decisions. For this reason, the general theory and principles of decision analysis have been summarized in the author's own words and focuses on decision-making under risk and is the most relevant situation to the topic researched. Numerous introductory worked examples have been included to improve understanding of decision theory and to provide a foundation for the new work provided in this thesis.

The main contribution of this thesis is the development of a decision theory process for making a mitigation decision on harmonic resonance in power systems.

The following new aspects are contributed to the field of harmonic analysis:

- A three-stage decision-making process, based on the scientific method of decision analysis, has been developed for making a mitigation decision on harmonic resonance in power systems. A quantitative decision model has been developed for conceptualizing and analyzing decision problems on the severity of harmonic resonance and mathematically relates the models major conceptual ingredients, decision alternatives (controllable inputs), states of nature (uncontrollable inputs) and result variables (outcomes) together.
- A methodology has been developed for identifying decision alternatives and is based on a "Resonance Frequency Band" and "Characteristic Resonance Frequency Band" approaches. Two – Controllable inputs are identified as decision alternatives, capacitor size and characteristic harmonic frequencies. Levels of electrical power demand for an end-user plant are identified as states of nature. A method of "relative heights" is used for assigning probabilities to the states of nature when the subjective method of assessment is applied. A deterministic decision model is developed as the states of nature have a likelihood of occurrence.
- A new decision table (table 4.4) is developed to structure and represent the harmonic resonance decision problem. An objective function, called the "Harmonic Resonance Severity Index" is developed to quantify the outcomes for the decision model and takes into account the two – controllable inputs and the states of nature. It also defines the severity of harmonic resonance.
- A process for making a decision on the non-monetary outcomes (%HRSI) of the decision model has been developed and is based on the scientific decision-making theory, called utility theory. In business, decision analysis is usually based on monetary outcomes and on the expected monetary rule (EMR). This was an

obstacle that had to be overcome as the %HRSI outcome is a non-monetary outcome and that the EMR rule did not apply. A new application for utility theory is introduced for making a decision on the size of capacitor to be installed. A “variable probability method” and an “elicitation session” have been introduced for deriving a utility function for a decision-maker who has to make a decision on the severity of harmonic resonance. With severities, the highest is the least preferred and the lowest the most desirable. It was therefore necessary to do research and to establish if a descending risk averse utility function was permissible in decision analysis. It was found that both ascending and descending utility functions are used in decision analysis. A utility function with a descending concave curve is introduced to take into account the “best” and “worst” severities.

- A formula, based on the “elicitation session” is introduced to calculate utility values for the % HRSI outcomes. A utility table and a constrained Expected Utility formula, which take into account the decision model’s major conceptual ingredients, are developed for making a decision (choice) on the size of the capacitor to be installed. This is the first decision step in the process and is called making “decision 1”.
- A new mitigation decision-making stage is developed and introduced as stage 3 in the decision theory process. Knowing only the severities that will result from the installation of the capacitor (decision1) is not sufficient for making a decision as to whether or not mitigation is needed. A further stage in the process had to be developed. For stage 3, a “Mitigation Index” has been developed and takes into account the major conceptual ingredients of the decision model and is applied to making a decision on whether or not mitigation is needed and is called making “decision 2”.
- Three case studies have been conducted to demonstrate the application of the newly developed decision theory process. This is the first time that decision analysis has been applied to making a decision on harmonic resonance and for making a mitigation decision.

8.2 CONCLUSIONS

- a. It has been found that decision analysis as a scientific tool can be applied for making decisions on harmonic resonance in power systems.
- b. It is found that the developed three-stage decision theory process is a rational methodology for conceptualizing, analyzing and solving decision problems involving mitigation of harmonic resonance.
- c. Before proceeding to investigate and make a decision it is found to be essential to develop a quantitative decision model, which must include a decision scenario and the identification of the problem and objectives. The model must also include three elements, decision alternatives, states of nature and a performance measure.

CHAPTER 8

It is also found that two controllable inputs, capacitor size and characteristic harmonics are required per decision alternative when making a decision on the severity of harmonic resonance.

- d. The new “objective function (%HRSI)” is found to be effective as it not only defines severity of harmonic resonance but also quantifies the outcome and takes into account the major conceptual ingredients of the decision model. This objective function is flexible in that it can be easily constrained within the ambit of a given decision problem.
- e. The new “Mitigation Index” is also found to be effective for making decision 2. The mitigation decisions made in this thesis, based on the %MI index have been confirmed by the IEEE standard loading indices for capacitors.
- f. It was found that the traditional decision theory process as applied to business and which used monetary outcomes and the “Expected Monetary Rule”, could not be used for making mitigation decisions. It was found that utility theory can be applied to non-monetary outcomes and that descending utility functions were possible and could be used to derive utility values. Therefore, the derived utility values could be used to develop a “Utility Table” and together with the “Expected Utility” value could be used as a technique to make decision 1.
- g. It was found that a meaningful and feasible mitigation decision is possible, despite the fact that the power system had a pre-existing resonance point and multiple resonance points arose after a new pf capacitor was installed.
- h. In cases 1 and 2 it was found that no mitigation is needed despite the fact that outcomes fall in the severe and very severe categories. Therefore, it is not always necessary to mitigate harmonic resonance.
- i. The “Resonance Frequency Band” combined with the “Characteristic Resonance Frequency Band” is an effective approach for constraining the number of decision alternatives. (Figures 5.10, 5.11, 6.3 to 6.6).
- j. When deriving utility functions, it is found that the risk neutral function can be useful to help a decision maker to find a probability indifference value when participating in an elicitation session and deriving a risk averse utility function.
- k. It is found that when identifying states of nature, it is a rule of thumb to obtain details of demand levels at minimum, mean and full load operating conditions. If no recordings of measurements are available then the states of nature need to be subjectively estimated so that the decision-making process becomes one of risk (deterministic) rather than one of uncertainty.
- l. Besides making a mitigation decision on the capacitor to be installed, it is found that other mitigation concerns need to be considered. The IEEE 519 and

IEC61000-2-4 standards provide limits for the VTHD% at the PCC and they need to be considered. Also the roles and responsibilities of all parties connected to a PCC need to be defined before implementing mitigation solutions.

- m. In case 3, it was found that mitigation was needed. The IEEE 519 limit of 5% was exceeded but IEC61000-2-4 limit of 8% was not. Three possible mitigation solutions were investigated, filters designed and their effects on mitigation were analyzed.
 - (i) Mitigation solution 1 involved the design and installation of a 2nd-order damped filter at the PCC. It was found that the filter is effective in that the new capacitor would not suffer damage and it also decreased the VTHD% below the IEEE 519 limit. Here the utility is responsible for the cost and upgrade provided the harmonic currents injected by end-user 3 are within the limits provided by the IEEE 519.
 - (ii) In solution 2, a notch filter is designed and applied in end-user 3's plant and it is found that the filter is also effective and the new capacitor would not be damped. The VTHD% at the PCC is also reduced to below 5%. Here end-user 3, the party injecting harmonics, is responsible for the cost and upgrade as the filter to be installed utilizes his capacitor bank.
 - (iii) A notch filter was designed for solution 3, for application in end-user 1's plant where the new capacitor is to be installed. This solution is the mitigating option when the IEC standard for the PCC is not surpassed. Here the end-user, who installs a new capacitor, is responsible for the cost and upgrade. The filter is found to be effective as no damage will be suffered by the new capacitor. The IEEE 519 limit was also reduced below 5%. Also, no damage would happen to the other capacitors in the system. The only drawback of this solution is that the filter needs a Q value of 50% higher than the recommended range.
- n. It is found that all three mitigation solutions considered would solve the problem in case 3 identified by the decision theory process. In terms of the IEEE 519, the utility and the end-user who injects harmonics are responsible parties for harmonic distortion in a system. The currents injected need to be checked against the IEEE 519 standard. If they are within the limits but the VTHD% at the PCC is greater than 5%, then the utility is responsible for the cost and upgrade.

8.3 RECOMMENDATIONS

The developed three-stage decision theory process is found to be a scientific tool that can be applied for making a mitigation decision on harmonic resonance in power systems.

Therefore, when a new power factor correction capacitor is to be installed in an end-user plant, it is recommended that this new process be applied.

8.4 FUTURE WORK

- a. More complex case studies need to be investigated. They should involve multiple harmonic sources at different voltage levels and at different locations.
- b. A case study should be conducted involving a series of power factor correction capacitors to be installed to a system having multiple end-users connected to a PCC. Here a decision theory process based on a tree diagram rather than a decision table should be investigated so that sequential decisions can be made on the mitigation of harmonic resonance as the system is expanded.
- c. Computer models need to be developed for active filters and they should be applied to solving mitigation problems identified by the developed decision theory process.

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APPENDIX 1

INTERNATIONAL STANDARDS (LIMITS) FOR PF CORRECTION CAPACITORS

The following is a summary of IEEE and IEC international standards (limits) relevant to pf correction capacitors.

1.1 CAPACITOR LIMITS

(i) Xu., W (2001) [3]

This paper states that when harmonic resonance exists, it does not imply that the capacitor will be damaged. It is therefore necessary to assess the severity of the resonance condition. To this end, standard loading indices and limits for shunt capacitors, as shown in the table below, are adopted.

STANDARD CAPACITOR LOADING INDICES AND LIMITS		
INDEX	EXPLANATION	LIMIT (%)
kvar	Apparent power of the capacitor $= (I_{RMS})(V_{RMS})$	135
V_{RMS}	RMS voltage of the capacitor	110
V_{peak}	Peak voltage of the capacitor	120
I_{RMS}	RMS current of the capacitor	180

Table A1.1

(ii) Dugan., R.C. (1996) [11]

The limits in Table A1.1 come from the “Recommended Practice for establishing capacitor capabilities when supplied by non-sinusoidal voltages” (IEEE Standard 18 - 1980).

An example of the application of these limits is given on p. 149 of the reference.

(iii) Wakileh, G.J. (2001) [5]

This textbook supports the IEEE Standard 18 -1980) (limits) but refers to a draft for a new standard, “Draft Standard for Shunt Power Capacitors, Standard p18 - 1999”

In regard to “Shunt Capacitors”, the standard states that capacitors can be continuously operated in a harmonic environment provided that:

(a) Reactive power does not exceed 135% of rating, i.e.

$$\frac{Q_c}{Q_{c1}} \leq 1.35 \quad (A1.1)$$

such that,

$$\frac{Q_c}{Q_{c1}} = \sum_{h=1} h \left(\frac{V_h}{V_1} \right)^2 = \sum_{h=1} \frac{1}{h} \left(\frac{I_h}{I_1} \right)^2 \leq 1.35 \quad (A1.2)$$

where: Q_c = the total reactive power delivered by the capacitor.

Q_{c1} = the reactive power delivered by the capacitor at the fundamental frequency.

h = the harmonic order, $h=1$ corresponds to the fundamental.

V_h = the h^{th} harmonic peak voltage.

I_h = the h^{th} harmonic peak current.

V_1 = peak voltage at fundamental frequency.

I_1 = peak current at fundamental frequency

- (b) Peak current does not exceed 180% (could be superseded to 130%) of rated peak current, i.e.

$$\frac{I_{\text{peak}}}{I_1} \leq 1.3 \quad (A1.3)$$

such that,

$$\frac{I_{\text{peak}}}{I_{1(\text{peak})}} = 1 + \text{CCF} \leq 1.3 \Rightarrow \text{CCF} \leq 0.3 \quad (A1.4)$$

where: CCF = Current Crest Factor.

NB!

$I_{\text{peak}}/I_{\text{RMS}} = \sqrt{2}$ is only true for case of pure sinusoid. Therefore, for non-sinusoidal waves, $I_{\text{peak}} \neq \sqrt{2} I_{\text{RMS}}$.

- (b) Peak voltages does not exceed 120% of rated peak voltage, i.e.

$$\frac{V_{\text{peak}}}{V_1} \leq 1.2 \quad (\text{A1.5})$$

such that,

$$\frac{V_{\text{peak}}}{V_1} = 1 + \text{VCF} \leq 1.2 \Rightarrow \text{VCF} \leq 0.2 \quad (\text{A1.6})$$

where: VCF = Voltage Crest Factor

(d) The root mean square voltage does not exceed 110% of rated voltage, i.e.

$$\frac{V_{\text{RMS}}}{V_1} \leq 1.1 \quad (\text{A1.7})$$

such that,

$$\frac{V_{\text{RMS}}}{V_1} = \sqrt{1 + (\text{VTHD})^2} \leq 1.1 \Rightarrow \text{VTHD} \leq \sqrt{0.21} = 45.8\% \quad (\text{A1.8})$$

Thus:

- 135% of rated reactive power (equations A1.1/A1.2)
- 180%(130%) of rated peak current (equations A1.3/A1.4)
- 120% of rated peak voltage (equations A1.5/A1.6)
- 110% of rated rms voltage (equations A1.7/A1.8)

The IEEE has a limit for rms current. Wakileh's textbook (p86) does not refer to a limit for rms current.

The limits for power (135%), peak voltage (120%) and rms voltage (110%) are the same as the IEEE. He includes rated peak current instead of rms current. Wakileh, suggests the limit for peak current could be reduced to 130% of rated peak current.

(iv) Gagaoudakis, N.G. (1998) [35]

This paper quotes the NEMA Standard (limits) for "Shunt Capacitors" and states that such capacitors can be applied within the following limitations including harmonic components.

- 135% of rated reactive power
- 110% of rated rms voltage
- 120% of rated peak voltage
- 180% of rated rms current.

These limits are the same as the IEEE limits (TableA1.1).

(v) IEC Standard (Limits)

Part 1 of IEC 60871 standard regulates capacitor units and banks used for power factor correction of ac power systems having a rated voltage above 1000V and frequencies of 15Hz to 60Hz. [36]. Section 4, clause 19 regulates overloads in terms of maximum permissible voltage levels, specifically long duration voltages. Clause 19.1 stipulates that capacitor banks/units shall be suitable for operation at voltage levels according to Table 6(IEC).

Type	Voltage factor x U_N (V r.m.s.)	Maximum duration	Observation
Power frequency	1.00	Continuous	Highest average value during any period capacitor energization . For energization periods less than 24 h exceptions apply as indicated below (see clause 28)
Power frequency	1.10	12h in every 24h	System voltage regulation and fluctuations
Power frequency	1.15	30 min in every 24h	System voltage regulation and fluctuations
Power frequency	1.20	5 min	Voltage rise at light load (see clause 28)
Power frequency	1.30	1 min	
Power frequency plus harmonics	Such that the current does not exceed the value given in clause 20 (see also clause 32 and clause 33)		

Table A1.2

The amplitudes of the over voltages that may be tolerated without significant deterioration of the capacitor depend on their duration, their total number and the capacitor temperature (see clause 28). It is assumed that the over voltages given in Table 6(IEC) and having a value higher than $1.15U_N$ do occur not more than 200 times in the capacitor's life.

Table 6(IEC) provides that when the type of voltage has a power frequency plus harmonics, then the current that flows must not exceed the value given in clause 20.

Clause 20 regulates maximum permissible current level whereas clause 19 regulates maximum permissible voltage levels.

Clause 20 states that capacitor units shall be suitable for continuous operation at an rms current of 1.30 times the current that occurs at rated sinusoidal voltage and rated frequency, excluding transients. Depending on the actual capacitor value, which may be a maximum of $1.15 C_N$, the maximum current can reach $1.5 I_N$ (see clause 32).

These "over current factors" are intended to take care of the combined effects due to harmonic and over voltages up to and including $1.10 U_N$ according to clause 19.1.

Clause 32 regulates “overload currents”. It states:

- a. Capacitors should never be operated with currents exceeding the permissible value specified in clause 20.
- b. The voltage waveform and the network characteristics should be determined before and after installing the capacitor. If sources of harmonics such as large rectifiers are present, special care should be taken.
- c. If the capacitor current should exceed the maximum value specified in clause 19.1, the pre-dominating harmonic should be determined in order to find the best remedy. This is where the usefulness of decision theory and the HRSI index can be seen.
- d. One or more of the following remedies may be effective in reducing the current:
 - (i) Moving some or all of the capacitors to other parts of the system.
 - (ii) Connection of a reactor in series with the capacitor to lower the resonant frequency of the circuit to a value below that of the disturbing harmonic (see clause 28).
 - (iii) Increasing the value of the capacitance where the capacitor is connected close to rectifiers.

Clause 28 refers to the choice of the rated voltage. It states:

- a. The rated voltage of the capacitor should not be less than the “maximum operating voltage” of the network to which the capacitor is to be connected.
- b. In certain networks, a considerable difference may exist between the operating and rated voltage of the network. This is of importance for capacitors, since their performance and life may be adversely affected by an undue increase of voltage across the capacitor dielectric.
- c. Where inductive elements are inserted in series with the capacitor to reduce the effects of harmonics, the resultant increase of the voltage at the capacitor terminals above the operating voltages of the network requires a corresponding increase in the rated voltage of the capacitor. If no information to the contrary is available, the operating voltage should be assumed equal to the rated or declared voltage of the network.

APPENDIX 1

When determining the voltage to be expected on the capacitor terminals, the following considerations should be taken into account:

- (i) When harmonics are present, capacitors are liable to operate at a higher voltage than that measured before connecting the capacitors.
- (ii) The voltage at the capacitor terminals may be particularly high at times of light load. In this case, the whole or part of the capacitor should be switched off to prevent overstressing of the capacitor and undue voltage increase in the network.

NB!

Clause 32 states that when the voltage rise at periods of light load is increased by capacitors, the saturation of transformer cores may be considerable. In this case, harmonics of abnormal magnitudes are produced, one of which may be amplified by resonance between transformer and capacitor. This is a further reason for recommending the disconnection of capacitors at periods of light load as stated above in clause 28.

SUMMARY

There are two standards for capacitor limits, IEEE and IEC. They can be compared as follows:

MAXIMUM HARMONIC LEVELS FOR SHUNT CAPACITORS CONTINUOUS OPERATION		
Index	IEEE Limit (%)	IEC Limit (%)
kvar	≤135	-
¹ I _{peak}	≤130	-
V _{peak}	≤120	-
V _{RMS}	≤110	-
² I _{RMS}	≤180	-
³ I _{RMS}		≤130 (clause 20)
¹ "Standard for Shunt Power Capacitors, standard 18 - 1992"		
² "Standard for Shunt Power Capacitors, standard 18 - 1980"		
³ "This over-current factor takes care of the combined effects due to harmonics and over voltages up to and including 1.10U _N "		

Table A1.3

The IEC standard, provides one limit only, namely I_{RMS} (130%), but states that this limit is intended to take care of the combined effects due to harmonics and over-voltages up to and including 1.10U_N.

APPENDIX 2

COMPUTER MODEL FOR SCAN STUDY – CASE 1

```
TITLE TITLE1="44 kV End-User Network"
      TITLE2="Consumer 1 only, Capacitor in service"
      TITLE3="Harmonic Resonance, 0.982 PF"
!
!
! Case : Harmonic Resonance case, Single-phase representation
!       of a three-phase network
!
!       Scan case
!
!       Solution will be for 1 Amp Injection
!       over a range of frequencies (60 Hz to 1200 Hz)
!
!
! -----SCAN SOURCE-----
!
SCAN NAME=SCAN1 BUS=HSOURCE1 FMIN=60 FMAX=1200 FINC=10 ANG=0.0
!
! -----VOLTAGE SOURCE-----
!
! Utility source, Positive Sequence Source
!
VSOURCE NAME=VSRC BUS=SRCV MAG=25403
!
!
! Positive Sequence Source Equivalent at 44 KVBUS
! Note: Impedance Value Given in Ohms at 60 Hz
!
BRANCH NAME=equiv FROM=SRCV TO=PCCBUS R=2.46 X=8.00
!
! -----CONSUMER 1-----
!
! 44 kV Distribution Line
! Note: Line capacitance not included due to short length
!
BRANCH NAME=LINE1 FROM=PCCBUS TO=BUS1 R=0.0581 X=1.2778
!
! Transformer at entrance to End-User
!
TRANSFORMER NAME = T1 MVA=10
H.1 = BUS1 X.1 = BUS1AA
kV.H = 44.00 kV.X = 4.16
%R.HX = 3.0 %X.HX = 15.0
%Imag=0 XRCONSTANT=YES
!
! Transformer connection to End-user Bus
!
BRANCH NAME=Con1 FROM=BUS1AA TO=BUS1AB R=0.0001 X=0.0
!
! Linear Load=100% Three-phase Motor Load
!
LINEARLOAD NAME=LOAD1 FROM=BUS1AB KVA=4500.0 KV=4.16 DF=0.80000
```

APPENDIX 2

```
%Parallel=0.0 %Series=100
!
! Capacitor connection to Consumer Bus
!
BRANCH NAME=R1 FROM=BUS1AB TO=BUS1AC R=0.0001 X=0.0
!
! Capacitor Bank
!
CAPACITOR NAME=CAP1 FROM=BUS1AC R=0.0 KV=4.16 MVA=2.025
!
! -----HARMONIC CURRENT SOURCE -----
!
! Metering element in series with 6-pulse drive
!
BRANCH NAME=rect1 FROM=BUS1AB TO=HSOURCE1 R=0.0001 X=0.0
!
! Three-Phase Harmonic Source
!
! 2.5745 MVA Drive, 6 Pulse, 4.16 kV, 6% commutation reactance
!
NONLINEARLOAD NAME=DRIVE1 BUS=HSOURCE1 KVA=858.156 KV=2.4018 DF=0.97107
TABLE=
{
{ 1, 357.30050, 166.19}, //
{ 5, 65.16396, 110.51},
{ 7, 42.53477, 82.08},
{ 11, 20.48435, 22.63},
{ 13, 14.32440, -9.39},
{ 17, 6.91028, -82.42},
{ 19, 4.96316, -125.51} //
}
!
!
RETAIN VOLTAGEs=yes
RETAIN CURRENTS=yes
!
!
! End of Input File
!
....
```

APPENDIX 3

COMPUTER MODEL FOR HARMONIC PENETRATION STUDY-CASE 1

```
TITLE TITLE1="44 kV End-User Network"
      TITLE2="Consumer 1 only, No Capacitor in service"
      TITLE3="Harmonic Penetration"
!
!
! Case : Harmonic Penetration case, Single-phase representation
!       of a three-phase network
!
!
! -----VOLTAGE SOURCE-----
!
! Utility source, Positive Sequence Source
!
VSOURCE NAME=VSRC BUS=SRCV MAG=25403
!
!
! Positive Sequence Source Equivalent at 44 KVBUS
! Note: Impedance Value Given in Ohms at 60 Hz
!
BRANCH NAME=equiv FROM=SRCV TO=PCCBUS R=2.46 X=8.00
!
! -----CONSUMER 1-----
!
! 44 kV Distribution Line
! Note: Line capacitance not included due to short length
!
BRANCH NAME=LINE1 FROM=PCCBUS TO=BUS1 R=0.0581 X=1.2778
!
! Transformer at entrance to End-User
!
TRANSFORMER NAME = T1 MVA=10
H.1 = BUS1 X.1 = BUS1AA
kV.H = 44.00 kV.X = 4.16
%R.HX = 3.0 %X.HX = 15.0
%Imag=0 XRCONSTANT=YES
!
! Transformer connection to End-user Bus
!
BRANCH NAME=Con1 FROM=BUS1AA TO=BUS1AB R=0.0001 X=0.0
!
! Linear Load=100% Three-phase Motor Load
!
LINEARLOAD NAME=LOAD1 FROM=BUS1AB KVA=4500.0 KV=4.16 DF=0.80000
%Parallel=0.0 %Series=100
!
! -----HARMONIC CURRENT SOURCE -----
!
! Metering element in series with 6-pulse drive
!
BRANCH NAME=rect1 FROM=BUS1AB TO=HSOURCE1 R=0.0001 X=0.0
!
! Three-Phase Harmonic Source
```


APPENDIX 3

```
!  
! 2.5745 MVA Drive, 6 Pulse, 4.16 kV, 6% commutation reactance  
!  
NONLINEARLOAD NAME=DRIVE1 BUS=HSOURCE1 KVA=858.156 KV=2.4018 DF=0.97107  
TABLE=  
{  
  { 1, 357.30050, 166.19}, //  
  { 5, 65.16396, 110.51},  
  { 7, 42.53477, 82.08},  
  {11, 20.48435, 22.63},  
  {13, 14.32440, -9.39},  
  {17, 6.91028, -82.42},  
  {19, 4.96316, -125.51} //  
}  
!  
!  
RETAIN VOLTAGEs=yes  
RETAIN CURRENTS=yes  
!  
!  
! End of Input File  
!  
.....
```

APPENDIX 4

CALCULATION OF OUTCOME VALUES FOR DECISION TABLE – CASE 1

Tune the pf correction capacitor in the end-user model to resonate at a characteristic harmonic [$fr=hch$], then determine the voltages $V_{C(sN)}[fr=hch]$ across the capacitor at the tuned frequency, using “Harmonic Penetration” studies” for a given state of nature.

$s_3 = (1.0)$ full load demand [capacitor set to full load (ω_1) value]

Following table is a summary of the results:

$s_3 = (1.0)$ full load demand		
Tuned to a hch	%QC(an) [Mvars]	$V_{C(s3)(ann)}[fr=hch]$ (V)
5	2.2460	536.296
7	1.1527	514.864
11	0.4685	413.537
13	0.3357	348.118
17	0.1964	225.255
19	0.1573	182.467

Table A4.1

$s_2 = (0.6)$ full load demand (Capacitor bank size remains unchanged).

$s_2 = (0.6)$ full load demand		
Tuned to a hch	%QC(an) [Mvars]	$V_{C(s3)(ann)}[fr=hch]$ (V)
5	2.2460	346.551
7	1.1527	323.936
11	0.4685	253.186
13	0.3357	211.437
17	0.1964	135.340
19	0.1573	109.270

Table A4.2

APPENDIX 4

$s_1 = (0.25)$ full load demand (capacitor bank size remains unchanged).

$s_1 = (0.25)$ full load demand		
Tuned to a hch	$\%Q_{C(an)} [Mvars]$	$V_{C(s_1)(ann)} [fr=hch] (V)$
5	2.2460	143.453
7	1.1527	130.076
11	0.4685	98.8171
13	0.3357	81.8804
17	0.1964	51.8792
19	0.1573	41.7502

Table A4.3

Set the pf correction capacitor size in the end-user model to $\%Q_{C(mpt-A)}$ and $\%Q_{C(mpt-B)}$, respectively. Then carry-out a “Harmonic Penetration” study” and calculate the $V_{C(sN)(ann)} [\%Q_{C(an)} [fr \neq hch]]$ values for a given state of nature.

The results for a_1 are:

$s_3 = (1.0)$ full load demand		
Decision Alternative 1		
Controllable input 1	Controllable input 2 _(ann)	$V_{C(s_3)(ann)} (V)$
$a_1 = 1.05682 \text{ Mvars}$ $\%Q_{C(mpt-A)}$ $fr \neq hch$	$a_{11} = 5$	166.787
	$a_{12} = 7$	458.820
	$a_{13} = 11$	50.502
	$a_{14} = 13$	24.499
	$a_{15} = 17$	7.589
	$a_{16} = 19$	4.666

Table A4.4

APPENDIX 4

$s_2 = (0.6)$ Full load demand		
Decision Alternative 1		
Controllable input 1	Controllable input 2 _(annm)	$V_{C(s2)(annm)} (V)$
$a_1 = 1.05682$ Mvars $\%Q_{C(mpt-A)}$ $fr \neq hch$	$a_{11} = 5$	114.501
	$a_{12} = 7$	333.515
	$a_{13} = 11$	30.256
	$a_{14} = 13$	14.911
	$a_{15} = 17$	4.674
	$a_{16} = 19$	2.881

Table A4.5

$s_1 = (0.25)$ Full load demand		
Decision Alternative 1		
Controllable input 1	Controllable input 2 _(annm)	$V_{C(s1)(annm)} (V)$
$a_1 = 1.05682$ Mvars $\%Q_{C(mpt-A)}$ $fr \neq hch$	$a_{11} = 5$	54.127
	$a_{12} = 7$	154.648
	$a_{13} = 11$	12.604
	$a_{14} = 13$	6.293
	$a_{15} = 17$	1.993
	$a_{16} = 19$	1.231

Table A4.6

The results for a_2 are:

$s_3 = (1.0)$ full load demand		
Decision Alternative 2		
Controllable input 1	Controllable input 2 _(annm)	$V_{C(s3)(annm)} (V)$
$a_2 = 1.7016$ Mvars $\%Q_{C(mpt-B)}$ $fr \neq hch$	$a_{11} = 5$	313.336
	$a_{12} = 7$	170.103
	$a_{13} = 11$	24.513
	$a_{14} = 13$	13.107
	$a_{15} = 17$	4.390
	$a_{16} = 19$	2.750

Table A4.7

APPENDIX 4

$s_2 = (0.6)$ Full load demand		
Decision Alternative 2		
Controllable input 1	Controllable input 2 _(anm)	$V_{C(s2)(anm)} (V)$
$a_2 = 1.7016 \text{ Mvars}$ $\%Q_{C(mpl-B)}$ $fr \neq hch$	$a_{11} = 5$	231.948
	$a_{12} = 7$	97.235
	$a_{13} = 11$	14.994
	$a_{14} = 13$	8.071
	$a_{15} = 17$	2.719
	$a_{16} = 19$	1.705

Table A4.8

$S_1 = (0.25)$ Full load demand		
Decision Alternative 2		
Controllable input 1	Controllable input 2 _(anm)	$V_{C(s1)(anm)} (V)$
$a_2 = 1.7016 \text{ Mvars}$ $\%Q_{C(mpl-B)}$ $fr \neq hch$	$a_{11} = 5$	118.631
	$a_{12} = 7$	39.000
	$a_{13} = 11$	6.355
	$a_{14} = 13$	3.440
	$a_{15} = 17$	1.165
	$a_{16} = 19$	0.731

Table A4.9

Now, calculating the outcomes:

$$\%HRSI_{(sN)(anm)} = \frac{V_{C(sN)(anm)} [\%Q_{C(an)} (fr \neq hch)]}{V_{C(sN)(anm)} [\%Q_{C(an)} (fr = hch)]} \times 100\% \quad (A4.1)$$

For example, using values from tables A4.4 and A4.1:

$$\%HRSI_{(s3)(5th)} = (166.787/536.296) \times 100\% = 31.09\%$$

In a similar manner all the other outcome values for the decision table can be calculated.

APPENDIX 5

CASE 1 - HARMONIC PENETRATION RESULTS CALCULATION OF MITIGATION INDICES AND COMPARISON TO INTERNATIONAL CAPACITOR LOADING STANDARDS

5.1 CAPACITOR VOLTAGE AND CURRENT RESULTS

(i) Low demand (s1)

Capacitor Voltage Results

Name	Freq	Fund	%THD	%RMS	%ASUM	RMSH	RMS
BUS1AC	60	2426.31	5.15572	100.133	106.979	125.094	2429.53

Name	Frequency	Magnitude	Angle
BUS1AC	1	2426.310	-1.965
BUS1AC	5	118.631	145.067
BUS1AC	7	39.000	-7.348
BUS1AC	11	6.355	-86.085
BUS1AC	13	3.440	-123.08
BUS1AC	17	1.165	155.100
BUS1AC	19	0.731	107.845

Capacitor Current Results

Name	Freq	Fund	%THD	%RMS	%ASUM	RMSH	RMS
CAP1	60	238.571	27.1467	103.619	141.813	64.764	247.205

Name	Frequency	Magnitude	Angle
CAP1	1	238.571	88.034
CAP1	5	58.322	-124.933
CAP1	7	26.843	82.651
CAP1	11	6.873	3.914
CAP1	13	4.398	-33.080
CAP1	17	1.947	-114.900
CAP1	19	1.367	-162.155

(ii) Mean demand (s2)

Capacitor Voltage Results

Name	Freq	Fund	%THD	%RMS	%ASUM	RMSH	RMS
BUS1AC	60	2345.62	10.7477	100.576	115.206	252.1	2359.13

Name	Frequency	Magnitude	Angle
BUS1AC	1	2345.62	-3.988

APPENDIX 5

BUS1AC	5	231.948	140.164
BUS1AC	7	97.235	-19.482
BUS1AC	11	14.994	-108.164
BUS1AC	13	8.071	-149.290
BUS1AC	17	2.719	120.753
BUS1AC	19	1.705	69.442

Capacitor Current Results

Name	Freq	Fund	%THD	%RMS	%ASUM	RMSH	RMS
CAP1	60	230.637	57.9816	115.594	193.318	133.727	266.601

Name	Frequency	Magnitude	Angle
CAP1	1	230.637	86.012
CAP1	5	114.033	-129.836
CAP1	7	66.925	70.517
CAP1	11	16.217	-18.164
CAP1	13	10.317	-59.290
CAP1	17	4.545	-149.247
CAP1	19	3.186	159.442

(iii) Full load demand (s3)

Capacitor Voltage Results

Name	Freq	Fund	%THD	%RMS	%ASUM	RMSH	RMS
BUS1AC	60	2256.64	15.8488	101.248	123.406	357.651	2284.81

Name	Frequency	Magnitude	Angle
BUS1AC	1	2256.640	-6.137
BUS1AC	5	313.336	133.641
BUS1AC	7	170.103	-31.819
BUS1AC	11	24.513	-131.611
BUS1AC	13	13.107	-177.135
BUS1AC	17	4.390	84.246
BUS1AC	19	2.750	28.624

Capacitor Current Results

Name	Freq	Fund	%THD	%RMS	%ASUM	RMSH	RMS
CAP1	60	221.888	88.4315	133.492	247.313	196.219	296.203

Name	Frequency	Magnitude	Angle
CAP1	1	221.888	83.862
CAP1	5	154.046	-136.359
CAP1	7	117.079	58.180

CAP1	11	26.513	-41.611
CAP1	13	16.754	-87.134
CAP1	17	7.338	174.246
CAP1	19	5.137	118.624

5.2 CALCULATION OF MITIGATION INDICES

a. Low demand (s₁)

$$\begin{aligned}\%MI_{[(82.69\%)(5th)]} &= \frac{247.205}{(1.30)(236.158)} \times 100\% \\ &= \frac{247.205}{307} \times 100\% \\ &= 80.52\% \quad (\%MI < 100\%) \end{aligned}$$

subject to: $V_{CI} \leq 1.10U_N$

$$\begin{aligned}V_{CI} &= \frac{2426.31}{2401.77} \times 100\% \\ &= 101\% \quad (V_{CI} < 1.10U_N) \end{aligned}$$

b. Mean demand(s₂)

$$\begin{aligned}\%MI_{[(66.93\%)(5th)]} &= \frac{266.601}{307} \times 100\% \\ &= 86.84 \quad (\%MI < 100\%) \end{aligned}$$

subject to: $V_{CI} \leq 1.10U_N$

$$\begin{aligned}V_{CI} &= \frac{2345.62}{2401.77} \times 100\% \\ &= 97.662\% \quad (V_{CI} \leq 1.10U_N) \end{aligned}$$

c. Full load demand (s₃)

$$\begin{aligned}\%MI_{[(58.42\%)(5th)]} &= \frac{296.203}{307} \times 100\% \\ &= 96.48\% \quad (\%MI < 100\%) \end{aligned}$$

APPENDIX 5

subject to: $V_{C1} \leq 1.10U_N$

$$V_{C1} = \frac{2256.64}{2401.77} \times 100\%$$

$$= 93.95\%$$

$$(V_{C1} \leq 1.10U_N)$$

5.3 CHECKING IEEE STANDARD LOADING INDICES FOR PF CAPACITORS

kvar Index

S_N	$V_{RMS}(V)$	$I_{RMS}(A)$	kvar	$Q_{C1}(ph)(kvar)$	$Q_{C1}(ph)/Q_{C1}(ph)(\%)$	Exceeds limit ($\leq 135\%$)
s_1	2429.53	247.205	600.59	567.2	105.8%	No
s_2	2359.13	266.601	628.95	567.2	110.8%	No
s_3	2284.81	296.203	676.76	567.2	119.3%	No

Table A5.1

Peak Current Index ($I_{l(peak)} = \sqrt{2} I_{lRMS(\omega 1)} = 236.158 \times \sqrt{2} = 333.97A$)

S_N	$I_{peak}(A)$	$I_{l(peak)}$	$I_{peak}/I_{l(peak)}(\%)$	Exceeds limits ($\leq 180\%$)
s_1	2429.53	247.205	107.52%	No
s_2	2359.13	266.601	151.4%	No
s_3	2284.81	296.203	179.0%	No

Table A5.2

Peak voltage Index ($V_{l(peak)} = \sqrt{2} V_{lRMS(\omega 1)} = 240.77 \times \sqrt{2} = 3396.6V$)

S_N	$V_{peak}(V)$	$V_{l(peak)}$	$V_{peak}/V_{l(peak)}(\%)$	Exceeds limit ($\leq 120\%$)
s_1	3352.63	3396.6	98.7%	No
s_2	3206.83	3396.6	94.4%	No
s_3	3059.15	3396.6	90.06%	No

Table A5.3

RMS Voltage Index

S_N	$V_{RMS}(V)$	$V_{l(RMS)}$	$V_{RMS}/V_{l(RMS)}(\%)$	Exceeds limit ($\leq 110\%$)
s_1	2429.53	2401.77	101.15%	No
s_2	2359.13	2401.77	98.22%	No
s_3	2284.81	2401.77	95.13%	No

Table A5.4

RMS Current Index

S_N	$I_{RMS}(A)$	$I_{I(RMS)}(A)$	$I_{RMS}/I_{I(RMS)}(\%)$	Exceeds limit ($\leq 180\%$)
S_1	247.205	236.158	104.67%	No
S_2	266.601	236.158	112.89%	No
S_3	296.203	236.158	125.42%	No

Table A5.5

APPENDIX 6

COMPUTER MODELS, DERIVATION OF UTILITY FUNCTION AND MITIGATION INDICES – CASE 2

6.1 BENCHMARK PARAMETERS FOR COMPUTER MODELS

a. Transformers

The ratings, impedance values and X/R ratio for the transformers are not disclosed in the paper [1]. Therefore, the ANSI standard was used as a guideline to assist with the selection of transformers.

Supply transformer (TS)

Using the disclosed loads to estimate the rating, TS is assumed to be 20MVA and have a X/R = 15.133 (ANSI guideline).

Let, $X_{(TS)} = 15.1 \Omega$, then $R = 15.1/15.133 = 1.0 \Omega$

End-user 1 (T1)

Assume, T1 to be 5MVA and have a X/R = 12.14(ANSI). Let, $X_{(T1)} = 12 \Omega$, then $R = 12/12.14 = 0.9885$

End-user 2 (T2)

Assume T2 to be 1MVA, X/R = 5.79(ANSI). Let, $X_{(T2)} = 5$, then $R = 0.8635 \Omega$

End-user 3 (T3A, T3B and T3C)

Here, 5MVA, 2MVA and 2MVA assumed and they have X/R ratios of 12.14, 7.098 and 7.098, respectively. Let, their X values be, 12, 7 and 7, respectively, thus,

$$R_{(T3A)} = 0.9885 \Omega, X_{(T3A)} = 12 \Omega$$

$$R_{(T3B)} = R_{(T3C)} = 0.9861 \Omega, X_{(T3A)} = 12 \Omega$$

b. Lines 2, 3 and 4

These lines are assumed to be short lines therefore capacitance is ignored.

c. Branches A, AA, AB, B, C and D

These branches are assumed to be very short, therefore values are assumed to be zero.

d. Loads 5A, 5B and 6B

These loads are linear loads.

e. Drive 1 and 2

These are 6-pulse drives ($hch = 6k \pm 1$, $k = 1, 2$ and 3 only). Drive 1 is rated at 2.1 MVA, 6kV, $d_{pf} = 0.8$ and Drive 2 is 0.25MVA, 0.4kV, $d_{pf} = 0.8$. The harmonic magnitudes are based on typical values determined from measurements, namely [11]:

$I_5 = (18\% I_1)$, $I_7 = (12\% I_1)$, $I_{11} = (6\% I_1)$, $I_{13} = (4\% I_1)$, $I_{17} = (2\% I_1)$ and $I_{19} = (1\% I_1)$.

f. Line 1(source impedance)

From trial and error it was found that the parameter values for “Line 1” should be $R_1 = 0.1872 \Omega$, $X_1 = 1.644 \Omega$ so that together with the other assumptions made for transformers and drives the desired results were obtained.

6.2 SCAN MODEL

When developing a scan model to conduct a frequency scan, the scan directive is employed. Constant current sources can be applied. They can be applied individually or simultaneously as a group.

The scan directive causes the “Superharm” program [37] to nullify all sources in the system. Voltage sources (VSOURCE) are short-circuited. Current sources (ISOURCE and NON-LINEARLOAD devices) are open circuited. When the Scan directive is used, 1 ampere is injected into an injection bus and its frequency is varied over a chosen range and Superharm calculates resultant driving point voltages ($V_D \propto Z_D$). The “Top” program (Output Processor Software Program) [37] is used for producing frequency scans and displaying the resonance curve, “Top” allows one or any combination of harmonic injections to be displayed. “Top” adds the solution for each selected group to obtain the total solution (frequency scan) and uses strict “linear superposition”.

The “Harmonic Scan Model” for this study is as follows:

```
TITLE TITLE1="40 kV End-User Network-Consumers 1, 2 and 3"
      TITLE2="PF Capacitor Banks at Buses 4A, 4B and 6BA"
      TITLE3="Harmonic Resonance"
!
!
! Case : Harmonic Resonance case, Single-phase representation
!       of a three-phase network
!
!       Scan case
!
!       Solution will be for 1 Amp Injection
!       over a range of frequencies (60 Hz to 1200 Hz)
!
! -----SCAN SOURCE GROUP-----
!
SCAN NAME=SCAN1 BUS=HSOURCE1 FMIN=60 FMAX=1200 FINC=10 ANG=0.0
SCAN NAME=SCAN2 BUS=HSOURCE2 FMIN=60 FMAX=1200 FINC=10 ANG=0.0
!
!
```

APPENDIX 6

```

!-----VOLTAGE SOURCE-----
!
! Utility source, Positive Sequence Source
!
VSOURCE NAME=VSRC BUS=BUS1 MAG=23094
!
!
! Positive Sequence Source Equivalent at 40 KVBUS
! Note: Impedance Value Given in Ohms at 60 Hz
!
BRANCH NAME=LINE1 FROM=BUS1 TO=BUS2 R=0.1872 X=1.1644
!
! Transformer at SOURCE
!
TRANSFORMER NAME = TS MVA=20
H.1 = BUS2 X.1 = BUS3
kV.H = 40.00 kV.X = 12.00
%R.HX = 1.0 %X.HX = 15.10
%Imag=0 XRCONSTANT=YES
!
!-----PF CORRECTION CAPACITORS AT BUS3-----
!
BRANCH NAME=BRANCHA FROM=BUS3 TO=BUS4 R=0.0001 X=0.0
!
! PF Capacitor Bank 4A connection
!
BRANCH NAME=BRANCHAA FROM=BUS4 TO=BUS4A R=0.0001 X=0.0
!
CAPACITOR NAME=CAP4A FROM=BUS4A KV=12.0 MVA=2.0
!
! PF Capacitor Bank 4B connection
!
BRANCH NAME=BRANCHAB FROM=BUS4 TO=BUS4B R=0.0001 X=0.0
!
CAPACITOR NAME=CAP4B FROM=BUS4B KV=12.0 MVA=2.0
!
!-----DISTRIBUTION NETWORK 1-----
!
! 12 kV Distribution Line
! Note: Line capacitance not included due to short length
!
BRANCH NAME=LINE2 FROM=BUS3 TO=BUS5 R=0.000001 X=0.000001
!
!-----CONSUMER 1-----
!
! Transformer at entrance to Consumer 1
!
TRANSFORMER NAME = T1 MVA=5
H.1 = BUS5 X.1 = BUS5A
kV.H = 12.00 kV.X = 6.00
%R.HX = 0.9885 %X.HX = 12.0
%Imag=0 XRCONSTANT=YES
!
! LinearLoad=100% Three-phase Motor Load
!
LINEARLOAD NAME=LOAD5A FROM=BUS5A KVA=2000.0 KV=6.00 DF=0.50000
%Parallel=0.0 %Series=100
!
!-----CONSUMER 2-----
!
! Transformer at entrance to Consumer 2
!
TRANSFORMER NAME = T2 MVA=1.0
H.1 = BUS5 X.1 = BUS5B
kV.H = 12.00 kV.X = 0.40
%R.HX = 0.8635 %X.HX = 5.0
%Imag=0 XRCONSTANT=YES
!
! LinearLoad=100% Three-phase Motor Load

```

APPENDIX 6

```

!
LINEARLOAD NAME=LOAD5B FROM=BUS5B KVA=250.0 KV=0.40 DF=0.90000
%Parallel=0.0 %Series=100
!
!-----DISTRIBUTION NETWORK 2-----
!
! 12 kV Distribution Line
! Note: Line capacitance not included due to short length
!
BRANCH NAME=LINE3 FROM=BUS3 TO=BUS6 R=0.000001 X=0.000001
!
!-----CONSUMER 3-----
!
! Metering element for monitoring injected harmonics from drives 1 & 2
!
BRANCH NAME=LINE4 FROM=BUS6 TO=BUS6A R=0.000001 X=0.000001
!
! Transformer feeding Drive1 in Consumer 3
!
TRANSFORMER NAME = T3A MVA=5.0
H.1 = BUS6A X.1 = BUS6AA
kV.H = 12.00 kV.X = 6.0
%R.HX = 0.9885 %X.HX = 12.0
%Imag=0 XRCONSTANT=YES
!
! Metering element in series with drive1
!
BRANCH NAME=BRANCHB FROM=BUS6AA TO=HSOURCE1 R=0.0001 X=0.0
!
! Three-Phase Harmonic Source 1
!
! 2.10 MVA Drive, 6 Pulse, 6.0 kV
!
NONLINEARLOAD NAME=DRIVE1 BUS=HSOURCE1 KVA=700.00 KV=3.4641 DF=0.80
TABLE=
{
{ 1, 202.07259, 166.19}, //
{ 5, 36.3706, 110.51},
{ 7, 24.2487, 82.08},
{ 11, 12.1243, 22.63},
{ 13, 8.0829, -9.39},
{ 17, 4.0414, -82.42},
{ 19, 2.0207, -125.51} //
}
!
! Transformer feeding Drive2 in Consumer 3
!
TRANSFORMER NAME = T3B MVA=2.0
H.1 = BUS6A X.1 = BUS6AB
kV.H = 12.00 kV.X = 0.40
%R.HX = 0.9861 %X.HX = 7.0
%Imag=0 XRCONSTANT=YES
!
! Metering element in series with drive2
!
BRANCH NAME=BRANCHC FROM=BUS6AB TO=HSOURCE2 R=0.0001 X=0.0
!
! Three-Phase Harmonic Source 2
!
! 0.25 MVA Drive, 6 Pulse, 0.4 kV
!
NONLINEARLOAD NAME=DRIVE2 BUS=HSOURCE2 KVA=83.333 KV=0.23094 DF=0.80
TABLE=
{
{ 1, 360.8426, 166.19}, //
{ 5, 64.9516, 110.51},
{ 7, 43.3011, 82.08},
{ 11, 21.6505, 22.63},
{ 13, 14.4337, -9.39},

```

APPENDIX 6

```
{ 17,    7.2168, -82.42},
{ 19,    3.6084, -125.51} //
}
!
! Transformer feeding pf corrected load in Consumer 3
!
TRANSFORMER  NAME = T3C  MVA=2.0
H.1 = BUS6  X.1 = BUS6B
kV.H = 12.00  kV.X = 0.40
%R.HX = 0.9861  %X.HX = 7.0
%Imag=0 XRCONSTANT=YES
!
! LinearLoad=100% Three-phase Motor Load in Consumer 3
!
LINEARLOAD NAME=LOAD6A FROM=BUS6B KVA=1000.0 KV=0.40 DF=0.80000
%Parallel=0.0 %Series=100
!
! PF Capacitor in Consumer 3
!
! Metering element in series with capacitor
!
BRANCH NAME=BRANCHD FROM=BUS6B TO=BUS6BA R=0.0001 X=0.0
!
CAPACITOR NAME=CAP6BA FROM=BUS6BA KV=0.40 MVA=0.30
!
-----RESULTS REQUIRED-----
!
RETAIN VOLTAGEs=yes
RETAIN CURRENTS=yes
!
! End of Input File
!
....
```

Operating Condition 1

40 kV End-User Network-Consumers 1, 2 and 3

PF Capacitor Banks at Buses 4A, 4B and 6BA

Harmonic Resonance

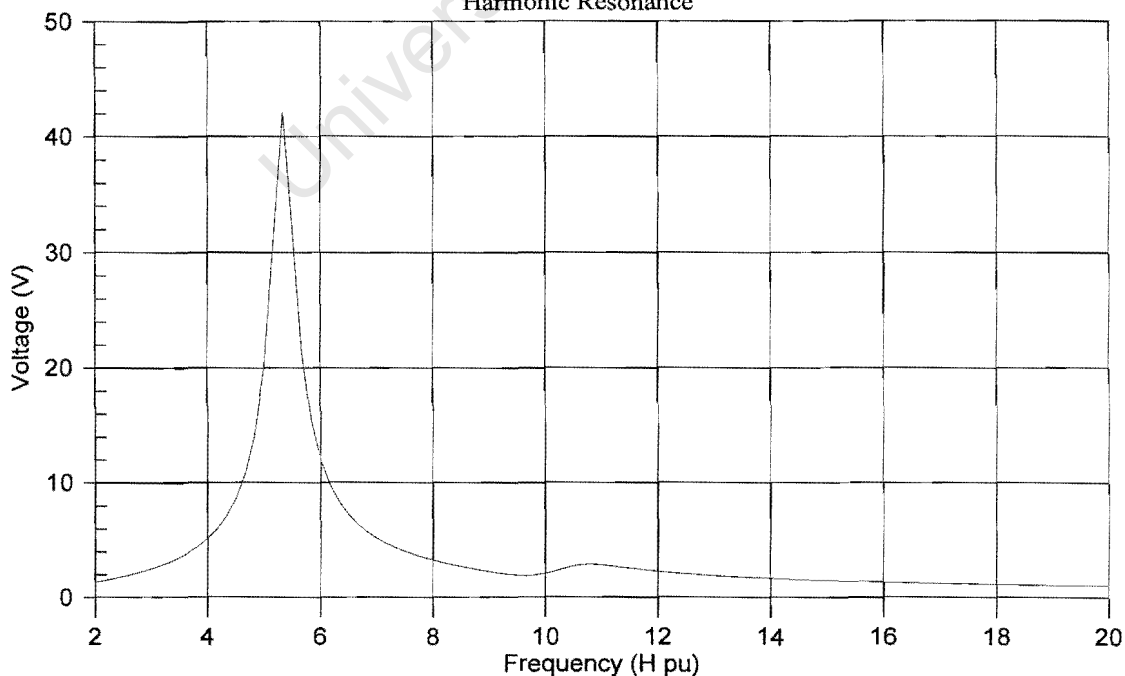


Figure A6.1 Resonance close to or at 5th harmonic

6.3 PENETRATION MODEL

Besides resonance frequency curves, magnitudes of voltages and currents are needed, therefore a computer model to carry out “Harmonic Penetration” is also needed and must also take into account one or more harmonic sources in the system. “Harmonic Penetration” studies are employed to calculate the voltages and currents at buses/nodes as a representation of a real system.

The “Harmonic Penetration Model” for this study is as follows:

```

TITLE TITLE1="40 kV End-User Network-Consumers 1, 2 and 3"
      TITLE2="PF Capacitor Banks at Buses 4A, 4B and 6BA"
      TITLE3="Harmonic Penetration"
!
!
! Case : Harmonic Penetration case, Single-phase representation
!   of a three-phase network
!
!
! -----VOLTAGE SOURCE-----
!
! Utility source, Positive Sequence Source
!
VSOURCE NAME=VSRC BUS=BUS1 MAG=23094
!
!
! Positive Sequence Source Equivalent at 40 KVBUS
! Note: Impedance Value Given in Ohms at 60 Hz
!
BRANCH NAME=LINE1 FROM=BUS1 TO=BUS2 R=0.1872 X=1.1644
!
! Transformer at SOURCE
!
TRANSFORMER NAME = TS MVA=20
H.1 = BUS2 X.1 = BUS3
kV.H = 40.00 kV.X = 12.00
%R.HX = 1.0 %X.HX = 15.10
%Imag=0 XRCONSTANT=YES
!
! -----PF CORRECTION CAPACITORS AT BUS3-----
!
BRANCH NAME=BRANCHA FROM=BUS3 TO=BUS4 R=0.0001 X=0.0
!
! PF Capacitor Bank 4A connection
!
BRANCH NAME=BRANCHAA FROM=BUS4 TO=BUS4A R=0.0001 X=0.0
!
CAPACITOR NAME=CAP4A FROM=BUS4A KV=12.0 MVA=2.0
!
! PF Capacitor Bank 4B connection
!
BRANCH NAME=BRANCHAB FROM=BUS4 TO=BUS4B R=0.0001 X=0.0
!
CAPACITOR NAME=CAP4B FROM=BUS4B KV=12.0 MVA=2.0
!
! -----DISTRIBUTION NETWORK 1-----
!
! 12 kV Distribution Line
! Note: Line capacitance not included due to short length
!
BRANCH NAME=LINE2 FROM=BUS3 TO=BUS5 R=0.000001 X=0.000001
!
! -----CONSUMER 1-----
!

```


APPENDIX 6

```

! Transformer at entrance to Consumer 1
!
TRANSFORMER  NAME = T1  MVA=5
H.1 = BUS5  X.1 = BUS5A
kV.H = 12.00  kV.X = 6.00
%R.HX = 0.9885  %X.HX = 12.0
%Imag=0 XRCONSTANT=YES
!
! LinearLoad=100% Three-phase Motor Load
!
LINEARLOAD NAME=LOAD5A FROM=BUS5A KVA=2000.0 KV=6.00 DF=0.50000
%Parallel=0.0 %Series=100
!
! -----CONSUMER 2-----
!
! Transformer at entrance to Consumer 2
!
TRANSFORMER  NAME = T2  MVA=1.0
H.1 = BUS5  X.1 = BUS5B
kV.H = 12.00  kV.X = 0.40
%R.HX = 0.8635  %X.HX = 5.0
%Imag=0 XRCONSTANT=YES
!
! LinearLoad=100% Three-phase Motor Load
!
LINEARLOAD NAME=LOAD5B FROM=BUS5B KVA=250.0 KV=0.40 DF=0.90000
%Parallel=0.0 %Series=100
!
! -----DISTRIBUTION NETWORK 2-----
!
! 12 kV Distribution Line
! Note: Line capacitance not included due to short length
!
BRANCH NAME=LINE3 FROM=BUS3 TO=BUS6 R=0.000001 X=0.000001
!
! -----CONSUMER 3-----
!
! Metering element for monitoring injected harmonics from drives 1 & 2
!
BRANCH NAME=LINE4 FROM=BUS6 TO=BUS6A R=0.000001 X=0.000001
!
! Transformer feeding Drive1 in Consumer 3
!
TRANSFORMER  NAME = T3A  MVA=5.0
H.1 = BUS6A  X.1 = BUS6AA
kV.H = 12.00  kV.X = 6.0
%R.HX = 0.9885  %X.HX = 12.0
%Imag=0 XRCONSTANT=YES
!
! Metering element in series with drive1
!
BRANCH NAME=BRANCHB FROM=BUS6AA TO=HSOURCE1 R=0.0001 X=0.0
!
! Three-Phase Harmonic Source 1
!
! 2.10 MVA Drive, 6 Pulse, 6.0 kV
!
NONLINEARLOAD NAME=DRIVE1 BUS=HSOURCE1 KVA=700.00 KV=3.4641 DF=0.80
TABLE=
{
{ 1, 202.07259, 166.19}, //
{ 5, 36.3706, 110.51},
{ 7, 24.2487, 82.08},
{11, 12.1243, 22.63},
{13, 8.0829, -9.39},
{17, 4.0414, -82.42},
{19, 2.0207, -125.51} //
}
!

```

APPENDIX 6

```

! Transformer feeding Drive2 in Consumer 3
!
TRANSFORMER NAME = T3B MVA=2.0
H.1 = BUS6A X.1 = BUS6AB
kV.H = 12.00 kV.X = 0.40
%R.HX = 0.9861 %X.HX = 7.0
%Imag=0 XRCONSTANT=YES
!
! Metering element in series with drive2
!
BRANCH NAME=BRANCHC FROM=BUS6AB TO=HSOURCE2 R=0.0001 X=0.0
!
! Three-Phase Harmonic Source 2
!
! 0.25 MVA Drive, 6 Pulse, 0.4 kV
!
NONLINEARLOAD NAME=DRIVE2 BUS=HSOURCE2 KVA=83.333 KV=0.23094 DF=0.80
TABLE=
{
{ 1, 360.8426, 166.19}, //
{ 5, 64.9516, 110.51},
{ 7, 43.3011, 82.08},
{ 11, 21.6505, 22.63},
{ 13, 14.4337, -9.39},
{ 17, 7.2168, -82.42},
{ 19, 3.6084, -125.51} //
}
!
! Transformer feeding pf corrected load in Consumer 3
!
TRANSFORMER NAME = T3C MVA=2.0
H.1 = BUS6 X.1 = BUS6B
kV.H = 12.00 kV.X = 0.40
%R.HX = 0.9861 %X.HX = 7.0
%Imag=0 XRCONSTANT=YES
!
! LinearLoad=100% Three-phase Motor Load in Consumer 3
!
LINEARLOAD NAME=LOAD6A FROM=BUS6B KVA=1000.0 KV=0.40 DF=0.80000
%Parallel=0.0 %Series=100
!
! PF Capacitor in Consumer 3
!
! Metering element in series with capacitor
!
BRANCH NAME=BRANCHD FROM=BUS6B TO=BUS6BA R=0.0001 X=0.0
!
CAPACITOR NAME=CAP6BA FROM=BUS6BA KV=0.40 MVA=0.30
!-----RESULTS REQUIRED-----
!
RETAIN VOLTAGEs=yes
RETAIN CURRENTS=yes
!
! End of Input File
!
....

```

6.3.1 VOLTAGE RESULTS (Bus 6A common to both drives)

Name	Frequency	Magnitude	Angle
BUS6A	1	6935.910	-1.775
BUS6A	5	791.707	36.723
BUS6A	7	136.244	171.817
BUS6A	11	37.106	16.740
BUS6A	13	16.557	-83.725

BUS6A	17	5.348	93.825
BUS6A	19	2.302	-3.070

6.3.2 CURRENT RESULTS

Name	Frequency	Magnitude	Angle
LINE3	1	149.057	-36.061
LINE3	5	26.757	146.043
LINE3	7	9.920	75.679
LINE3	11	8.457	-74.476
LINE3	13	4.891	-174.516
LINE3	17	2.256	3.403
LINE3	19	1.111	-93.400

6.4 RESULTS AT END-USER 1 TO CONFIRM DISPLACEMENT POWER FACTOR

Name	Frequency	Magnitude	Angle
BUS5A	1	3322.59V	-2.905
LOAD5A	1	184.58A	-62.90

6.5 %HRSI CALCULATIONS

(1) Tuning to 5th and 7th harmonic resonance.

5 th AND 7 th HARMONIC TUNING CAPACITOR VOLTAGE RESULTS (s ₁)			
Q _{C(an)} (fr = hch)	V _{C(s1)(anm)} [fr = hch]		Cap5AA tuning
433.013kvars	a ₂₁	1035.06V	5 th
1399.037kvars	a ₂₂	221.72V	7 th

Table A6.1 5th and 7th Harmonic tuning capacitor voltage results (s₁)

5 th AND 7 th HARMONIC TUNING CAPACITOR VOLTAGE RESULTS (s ₂)			
Q _{C(an)} (fr = hch)	V _{C(s2)(anm)} [fr = hch]		Cap5AA tuning
433.013kvars	a ₂₁	988.076V	5 th
1399.037kvars	a ₂₂	227.595V	7 th

Table A6.2 5th and 7th Harmonic tuning capacitor voltage results (s₂)

5 th AND 7 th HARMONIC TUNING CAPACITOR VOLTAGE RESULTS (s ₃)			
Q _{C(an)} (fr = hch)	V _{C(s3)(anm)} [fr = hch]		Cap5AA tuning
433.013kvars	a ₂₁	917.977V	5 th
1399.037kvars	a ₂₂	234.799V	7 th

Table A6.3 5th and 7th Harmonic tuning capacitor voltage results (s₃)

(2) Tuning to % $Q_{C(mpt-A)}$ and % $Q_{C(mpt-B)}$ values

% $Q_{C(mpt-A)}$ AND % $Q_{C(mpt-B)}$ CAPACITOR VOLTAGE RESULTS (s_1)				
	$Q_{C(mpt)}$	$V_{C(s1)(anm)} [fr \neq hch]$		Cap5AA tuning
A	1323.403kvars	$a_{21}(5^{th})$	239.804V	a_2
		$a_{22}(7^{th})$	301.192V	
B	1565.543kvars	$a_{31}(5^{th})$	190.084V	a_3
		$a_{32}(7^{th})$	273.989V	

Table A6.4 % $Q_{C(mpt-A)}$ and % $Q_{C(mpt-B)}$ capacitor voltage results (s_1)

% $Q_{C(mpt-A)}$ AND % $Q_{C(mpt-B)}$ CAPACITOR VOLTAGE RESULTS (s_2)				
	$Q_{C(mpt)}$	$V_{C(s1)(anm)} [fr \neq hch]$		Cap5AA tuning
A	1323.403kvars	$a_{21}(5^{th})$	246.754V	a_2
		$a_{22}(7^{th})$	294.445V	
B	1565.543kvars	$a_{31}(5^{th})$	194.258V	a_3
		$a_{32}(7^{th})$	277.273V	

Table A6.5 % $Q_{C(mpt-A)}$ and % $Q_{C(mpt-B)}$ capacitor voltage results (s_2)

% $Q_{C(mpt-A)}$ AND % $Q_{C(mpt-B)}$ CAPACITOR VOLTAGE RESULTS (s_3)				
	$Q_{C(mpt)}$	$V_{C(s1)(anm)} [fr \neq hch]$		Cap5AA tuning
A	1323.403kvars	$a_{21}(5^{th})$	255.318V	a_2
		$a_{22}(7^{th})$	286.323V	
B	1565.543kvars	$a_{31}(5^{th})$	199.351V	a_3
		$a_{32}(7^{th})$	280.557V	

Table A6.6 % $Q_{C(mpt-A)}$ and % $Q_{C(mpt-B)}$ capacitor voltage results (s_3)

(3) Calculate % $HRSI_{(sN)(anm)}$ values

% $HRSI_{(s1)(a21)}$	=	239.804/1035.06	=	23.168%	least severe
% $HRSI_{(s1)(a22)}$	=	301.192/221.728	=	135.838%	very severe
% $HRSI_{(s1)(a31)}$	=	190.084/1035.06	=	18.364%	least severe
% $HRSI_{(s1)(a32)}$	=	273.989/221.728	=	123.569%	very severe
% $HRSI_{(s2)(a21)}$	=	246.754/988.076	=	24.973%	least severe
% $HRSI_{(s2)(a22)}$	=	294.445/227.595	=	129.372%	very severe
% $HRSI_{(s2)(a31)}$	=	194.258/988.076	=	19.660%	least severe
% $HRSI_{(s2)(a32)}$	=	277.273/227.595	=	119.630%	very severe
% $HRSI_{(s3)(a21)}$	=	255.318/917.977	=	27.813%	least severe
% $HRSI_{(s3)(a22)}$	=	286.323/234.799	=	121.943%	very severe

APPENDIX 6

$$\%HRSI_{(s3)(a31)} = 199.351/917.977 = 21.716\% \quad \text{least severe}$$

$$\%HRSI_{(s3)(a32)} = 280.557/234.799 = 119.488\% \quad \text{very severe}$$

6.6 VARIABLE PROBABILITY METHOD AND UTILITY FUNCTION

a. Identify the worst and the best severity of resonance from the decision table; namely:

$$135.838\% \text{ (worst)}, \quad 119.488\% \text{ (best)}$$

b. Rank the severity of resonance outcomes from best to worst, namely:

$$\begin{aligned} &119.448\% \text{ (best)} \\ &119.630\% \\ &121.943\% \\ &123.569\% \\ &129.372\% \\ &135.838\% \text{ (worst)} \end{aligned}$$

c. Assign a utility value to the best and worst severities, that is, assign a value of “1” to the best and “0” to the worst, so:

$$\begin{aligned} U(119.448\%) &= 1.0 \\ U(135.838\%) &= 0.0 \end{aligned}$$

d. Apply the variable probability method to determine the utility values for the other severities that are not yet known.

(i) Assigning a utility value to 119.630% using the elicitation session defined in table 4.5 (chapter 4).

Question: Which “p” value would make you indifferent between “A” and “B”?

A: 119.630% severity for certain, or

B: a “p” chance of obtaining 119.448% (best outcome) or
a “1-p” chance of obtaining 135.836% (worst outcome).

A series of questions is asked:

If $p=0.5$ would you be indifferent?

Answer: No, I would not be indifferent, I would choose “A” because “B” would only give me a 50% chance of obtaining the best outcome but it would also give a 50% chance of obtaining the worst outcome.

APPENDIX 6

The game must therefore be made more attractive by increasing “p”. As 119.63% is very close to the best outcome, then using the risk neutral utility function as a guideline and visualizing a concave curve ask the question,

If $p = 0.985$ would you be indifferent?

Answer: Yes, I would be indifferent. Both “A” and “B” would be equally attractive. “A” would give 119.63% severity for certain (sure) and “B” would give a high chance of obtaining the best outcome and only “ $1-0.985=0.015$ ” chance of obtaining the worst outcome. The risk is only 1.5%. If $p > 0.985$, say 0.99, I would not be indifferent and would choose “B” as there would be a 99% chance of obtaining the best outcome and only a 1% chance of obtaining the worst outcome and this would lead to a result better than 119.63%.

Therefore $p^* = 0.985$

Now, calculate the utility for 119.63% using the formula (equation 4.23 chapter 4):

$$U(119.63\%) = 0.985(1.0) + (1-0.015)(0) = 0.985$$

(ii) Assigning a utility value to 121.943%

Question: Which “p” value would make you indifferent between “A” and “B”?

A: 121.943% severity for certain (sure), or

B: a “p” chance of obtaining 119.448% (best outcome), or

a “ $1-p$ ” chance of obtaining 135.836% (worst outcome).

If after a series of questions with different “p” values have been asked, (e.g., $p = 0.5$, $p = 0.6$, etc), the game being made more attractive, and using the risk neutral curve as a guideline and visualizing a concave curve, the question is asked,

If $p = 0.96$ (greater than 0.95 on risk neutral curve), would you be indifferent?

Answer: Yes, I would be indifferent. Both “A” and “B” would be equally attractive. “A” would give 121.943% severity for certain (sure) and “B” would give a high chance of obtaining the best outcome and only a $1-0.96 = 0.04$ chance of obtaining the worst outcome. The risk is only 4%. If $p > 0.96$, say $p = 0.98$, I would not be indifferent and would choose “B” as there would be a 98% chance of obtaining the best outcome and only a 2% chance for the worst outcome and this would lead to a result better than 121.943%, that is, I would have a very good chance of obtaining 119.448% (best) severity and the gamble would be worth the risk.

Therefore, $p^* = 0.96$ and $U(121.943\%) = 0.96$

This agrees with the rule, the more desirable an outcome the higher the utility will be and therefore vice versa, the less desirable an outcome the lower the utility will be. As 121.943% is less desirable than 119.63% it must have a lower utility value, $0.96 < 0.985$.

(iii) Assign a utility value to 123.569%

Question: Which “p” value would make you indifferent between “A” and “B”?

- A: 123.569% severity for certain (sure), or
 B: a “p” chance of obtaining 119.448% (best outcome), or
 a “1-p” chance of obtaining 135.836% (worst outcome).

If after a series of questions with different “p” values (e.g., $p = 0.93$, $p = 0.9$ etc), the game being made less attractive, and using the risk neutral curve, the question is asked,

If $p = 0.8$ (greater than 0.75 on risk neutral curve), would you be indifferent?

Answer: Yes, I would be indifferent. Both “A” and “B” would be equally attractive to me. “A” would give 123.569% for certain (sure) and “B” would give a good chance of obtaining the best outcome and only a $1 - 0.8 = 0.2$ chance of obtaining the worst outcome. The risk is only 20%. If $p > 0.8$, say $p = 0.9$, I would not be indifferent and would choose “B” as there would be a 90% chance of obtaining the best outcome and only a 10% chance of the outcome and thus would lead to a result better than 123,569%, that is, I would have a very good chance of obtaining 119.488% (best) severity and the gamble would be worth the risk.

Therefore, $p^* = 0.8$ and $U(123.569\%) = 0.8$.

(iv) Assign a utility value to 129.372%

Question: Which “p” value would make you indifferent between “A” and “B”?

- A: 129.372% severity for certain, or
 B: a “p” chance of obtaining 119.448% (best outcome) or
 a “1-p” chance of obtaining 135.836% (worst outcome).

If after a series of questions with different “p” values (less or more attractive), and using the risk neutral as a guideline and visualizing a concave curve, the question is asked,

If $p = 0.6$ (greater than 0.4 on risk neutral curve), would you be indifferent?

Answer: Yes, I would be indifferent. Both “A” and “B” would be equally attractive to me. “A” would give 129.732% severity for certain (sure) and “B” would give a fair chance of obtaining the best outcome but would increase the risk to $1 - 0.6 = 0.4$, a 40% chance of obtaining the worst outcome. If $p > 0.6$, say 0.7, I would not be indifferent and would choose “B” as there would be a 70% chance for the best and only a 30% chance for the worst outcomes, respectively. This could lead to a result better than 129.372% and would be worth

the risk even if the worst was obtained (135.836%) as this would not be very much worse than the 129.372% both being less desirable.

Therefore, $p^* = 0.6$ and $U(129.372) = 0.6$

(v) As all the utility values have been determined, now plot the utility curve for the decision maker together with the risk neutral curve, namely:

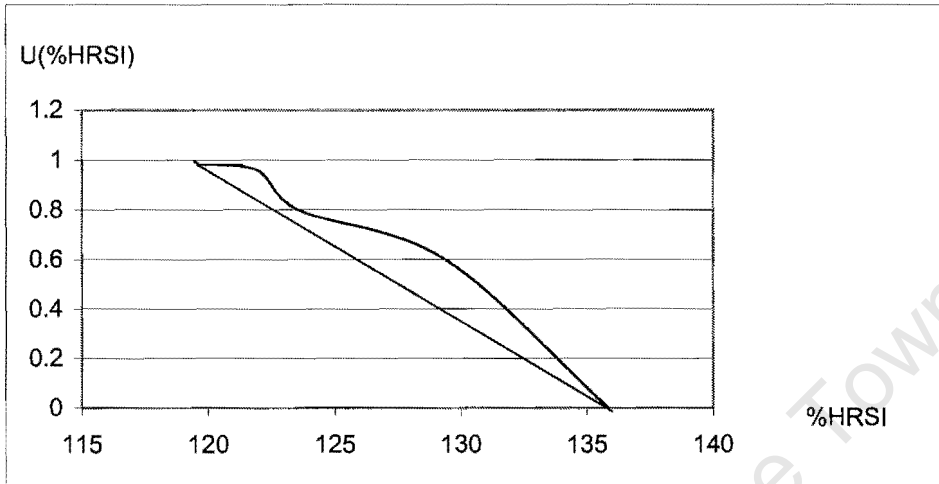


Figure A6.2 Utility function for decision-maker – case 2

The utility values are:

$$\begin{aligned}
 U(119.448\%) &= 1.000 \\
 U(119.630\%) &= 0.985 \\
 U(121.943\%) &= 0.960 \\
 U(123.569\%) &= 0.800 \\
 U(129.372\%) &= 0.600 \\
 U(135.836\%) &= 0.000
 \end{aligned}$$

The concave graph is an estimate of the decision-makers utility function for this decision problem. Note the shape of the utility function is “risk averse”.

6.7 CALCULATION OF MITIGATION INDICES

(i) 0.25 full load demand

$$\begin{aligned}
 \%MI_{[(123.569\%)(7^{th})]} &= \frac{184.062}{(1.30)(150.6445)} \times 100\% \\
 &= \frac{184.062}{195.8378} \times 100\% \\
 &= 93.986\% \quad (\%MI < 100\%)
 \end{aligned}$$

APPENDIX 6

subject to: $V_{Cl} \leq 1.10U_N$

$$\begin{aligned} V_{Cl} &= \frac{3650.65}{3464.1016} \times 100\% \\ &= 105.385\% (V_{Cl} < 1.10U_N) \end{aligned}$$

(ii) 0.6 full load demand

$$\begin{aligned} \%MI_{[(119.630\%)(7^{th})]} &= \frac{181.852}{195.8378} \times 100\% \\ &= 92.858\% (\%MI < 100\%) \end{aligned}$$

subject to: $V_{Cl} \leq 1.10U_N$

$$\begin{aligned} V_{Cl} &= \frac{3573.64}{3464.1016} \times 100\% \\ &= 103.162\% (V_{Cl} < 1.10U_N) \end{aligned}$$

(iii) 1.0 full load demand

$$\begin{aligned} \%MI_{[(119.488\%)(7^{th})]} &= \frac{179.461}{195.8378} \times 100\% \\ &= 91.637\% (\%MI < 100\%) \end{aligned}$$

subject to: $V_{Cl} \leq 1.10U_N$

$$\begin{aligned} V_{Cl} &= \frac{3489.14}{3464.1016} \times 100\% \\ &= 100.722\% (V_{Cl} < 1.10 U_N) \end{aligned}$$

APPENDIX 7

CALCULATION OF MITIGATION INDICES - HARMONIC FILTER THEORY AND COMPUTER MODEL INCLUDING 2ND - ORDER DAMPED FILTER AT PCC FOR CASE 3

7.1 CALCULATION OF MITIGATION INDICES

i) 0.25 full load demand

$$\begin{aligned}\%MI_{[(123.569\%)(7^{th})]} &= \frac{173.729}{(1.30)(127.44)} \times 100\% \\ &= \frac{173.729}{165.672} \times 100\% \\ &= 104.86\% \quad (\%MI > 100\%) \end{aligned}$$

subject to: $V_{C1} \leq 1.10U_N$

$$\begin{aligned}V_{C1} &= \frac{3622.29}{3464.1016} \times 100\% \\ &= 104.56\% \quad (V_{C1} < 1.10U_N) \end{aligned}$$

(ii) 0.6 full load demand

$$\begin{aligned}\%MI_{[(119.630\%)(7^{th})]} &= \frac{169.574}{165.672} \times 100\% \\ &= 102.355\% \quad (\%MI > 100\%) \end{aligned}$$

subject to: $V_{C1} \leq 1.10U_N$

$$\begin{aligned}V_{C1} &= \frac{3547.69}{3464.1016} \times 100\% \\ &= 102.41\% \quad (V_{C1} < 1.10U_N) \end{aligned}$$

(iii) 1.0 full load demand

$$\begin{aligned}\%MI_{[(119.488\%)(7^{th})]} &= \frac{165.044}{165.672} \times 100\% \\ &= 99.62\% \quad (\%MI < 100\%) \end{aligned}$$

subject to: $V_{Cl} \leq 1.10U_N$

$$V_{Cl} = \frac{3465.71}{3464.1016} \times 100\%$$

$$= 100.046\% (V_{Cl} < 1.10 U_N)$$

7.2 HARMONIC FILTER THEORY

7.2.1 SERIES TUNED FILTER

The use of a passive filter can reduce harmonic resonance. The simplest is a series combination of a reactor and a capacitor and is usually tuned to one of the low characteristic harmonics in a system. At the tuned harmonic the capacitor and reactor reactance values are the same and the filter is purely resistive [5].

Let Q_C = capacitor size in Mvars at “ ω_1 ”, then

$$X_C = \frac{kV^2}{Q_C} \quad (A7.1)$$

To trap a certain characteristic harmonic $[hch_{(n)}]$, then:

$$[hch_{(n)}]X_L = \frac{X_C}{[hch_{(n)}]} \quad (A7.2)$$

$$X_L = \frac{X_C}{[hch_{(n)}]^2} \quad (A7.3)$$

The filters characteristic impedance is:

$$X_{hch(n)} = \sqrt{X_L X_C} = \sqrt{\frac{L}{C}} \quad (A7.4)$$

Let, Q = quality factor of the filter and fall in the range [5],

$$30 < Q < 100 \quad (A7.5)$$

The reactor resistance is found as follows [11]:

$$R = \frac{X_{hch(n)}}{Q} \quad (A7.6)$$

For the filter, the impedance at any hch , is:

$$Z_F(hch) = R + j \left[hchX_L - \frac{X_C}{hch} \right] \quad (A7.7)$$

For example: If $X_C = 507 \Omega$ and the series tuned filter is tuned to the $hch(13)$, then,

$$X_L = \frac{X_C}{[hch_{(n)}]^2} = \frac{507}{13^2} = 3\Omega$$

If $f_r = 60\text{Hz}$, $L = 3/2 \pi 60 = 7.9577\text{mH}$
 $C = 1/2 \pi 60507 = 5.2319\mu\text{F}$

The filter characteristic impedance is:

$$X_{hch(n)} = \sqrt{X_L X_C} = \sqrt{3 \times 507} = 39\Omega$$

or,

$$X_{hch(n)} \sqrt{\frac{L}{C}} = \sqrt{\frac{7.9577 \times 10^{-3}}{5.2319 \times 10^{-6}}} = 39\Omega$$

Assuming a quality factor $Q = 100$, the reactor resistance is:

$$R = \frac{39}{100} = 0.39\Omega$$

Therefore, the elements of the filter are: $R = 0.39$, $L = 7.9577\text{mH}$, $C = 5.2319\mu\text{F}$

$$Z_{(13th)} = 0.39 + j[(13 \times 3) - (507 \div 13)] = 0.39 + j0 = 0.39\Omega = R$$

7.2.2 NOTCH FILTER

The “notch” filter is the most commonly employed type of passive filter. It has the same series elements but differs from a series tuned filter in that it is a detuned filter. It is tuned below a characteristic harmonic where it absorbs some of the harmonic but not as much as a series tuned filter, which is tuned to a characteristic harmonic. They are applied as shunt passive filters and divert harmonic current away from a normal flow path.

The capacitor used in a notch filter is typically a pf correction capacitor in an end-user plant. Notch filters can therefore provide power factor correction, but at the same time suppress harmonics [30].

APPENDIX 7

In notch filters capacitors are tuned with series reactors to avoid severities of resonance rather than to reduce harmonics.

The resonance frequency “ f_{notch} ” is chosen to be safely away from any characteristic harmonic. This safety margin is used in case a system parameter changes and to prevent the change from moving the “ f_{notch} ” point so that it coincides to a “ f_{ch} ”.

To avoid characteristic harmonic resonance, notch filters are added to the system starting with the lowest “ f_{ch} ” found in the system.

In utility distribution systems, pf capacitors are usually star-connected. If the system suffers from imbalance, a neutral can be provided so that zero sequence triplen harmonics can pass.

Notch filters are best applied at buses in a system where the short-circuit reactance can be expected to remain unchanged.

When notch filters are applied to a simple network, two resonance frequencies are identified [41]:

- a. A series resonance frequency for the filter itself,

$$f_s = \frac{1}{2\pi\sqrt{L_f C_f}} \quad (\text{A7.8})$$

where: L_f and C_f are the inductance and capacitance in the filter.

- b. A parallel resonance frequency as the filter together with the supply path (L_s) form a parallel circuit and the tuned frequency will be lower than f_s ,

$$f_p = \frac{1}{2\pi\sqrt{(L_f + L_s)C_f}} \quad (\text{A7.9})$$

It is recommended to shift f_s to a value of 3% to 10% below the lowest-order characteristic harmonic frequency (f_{lowest}) produced by the harmonic source in the system, namely:

Let x represent the fraction used for shifting f_s below f_{lowest} , then

$$f_{\text{notch}} = f_{\text{lowest}} - [(x)(f_{\text{lowest}})] \quad (\text{A7.10})$$

Where: $0.03 \leq x \leq 1.0$ (A7.11)

The procedure for designing a notch filter is as follows: [42]

APPENDIX 7

1. Determine:
$$X_c = \frac{kV^2}{Q_{C(rated)}} \quad (A7.12)$$

where: $Q_{C(RATED)}$ could be the pf correction capacitor size chosen by the developed decision theory process or any other capacitor in a system to be used as part of a notch filter.

2. Determine the “C” value from X_c for the given fundamental frequency (f_1).

3. Determine the notch frequency to which the filter is to be tuned by selecting a value for “x”.

$$h_{notch} = f_{lowest} - [(x)(f_{lowest})] \quad (A7.13)$$

4. Using, equation A7.9 but substituting h_{notch} for f_s calculate the inductance (L_f) value for the reactor to be connected in series with the capacitor.

$$L_f = \frac{1}{(2\pi h_{notch})^2 C_f} \quad (A7.14)$$

5. Determine the inductive reactance value for the reactor (X_R), at fundamental frequency (f_1).

$$X_R = 2 \pi f_1 L_f \quad (A7.15)$$

Alternatively,
$$X_R = \frac{X_c}{(h_{notch})^2} \quad (A7.16)$$

6. Choose a Q factor, then determine the resistance for the reactor (R_R) [11].

$$R_R = \frac{(h_{notch})X_L}{Q} \quad (A7.17)$$

9. Rate the reactor at 110% of the current it draws at f_1 .

For example, [11], [30]: Let $Q_C = 0.5Mvars$, $kV = 0.48$, $hch = 6k \pm 1$, $k = 1, 2, 3, \dots n$.

1.
$$X_c = \frac{0.48^2}{0.5} = 0.4608\Omega$$

2. Let $f_1 = 60Hz$, $C = 5756.472\mu F$

3. Let $x = 0.06$ (6%)

$$h_{\text{notch}} = 300 - [(0.06)(300)] = 282\text{Hz (4.7}^{\text{th}})$$

$$4. \quad L_f = \frac{1}{(2\pi 282)^2 5756.422 \times 10^{-6}} = 0.05533\text{mH}$$

$$5. \quad X_R = 2\pi(60)(0.05533 \times 10^3) = 0.02086\Omega$$

$$\text{or,} \quad X_R = \frac{0.4608}{4.7^2} = 0.02086\Omega$$

6. Using equation 7.17 as a guideline let $Q = 30$, then,

$$R_R = \frac{(282)(0.02086)}{30} = 0.1961\Omega$$

7.2.3 2ND - ORDER DAMPED FILTER

A 2nd – order damped filter has a capacitor in series with a parallel combination of a resistor and a reactor. Its function is to offer a low impedance path to a constrained range of frequencies.

It is often used as a high pass filter. It then provides a low impedance path for high order harmonics [e.g., $h_{ch(n)} > 17^{\text{th}}$] but stopping low order harmonics [$h_{ch(n)} < 17$]. Their quality factor range is usually:

$$0.5 < Q < 5 \quad (\text{A7.18})$$

Damped filters are usually not tuned to $h_{ch(n)}$ values, e.g., 10.7, 16.5, etc.

To design this filter, the following steps are followed [5].

Step 1: Let the 2nd – order damped filter be tuned to h_n harmonic.

Step 2: Let Q_C be the capacitor size in Mvars to be used in the filter.

$$\text{Step 3: Determine, } X_C = \frac{kV^2}{Q_C} \quad (\text{A7.19})$$

Step 4: Determine the reactance of the reactor at fundamental frequency to trap h_n ,

$$X_L = \frac{X_C}{h_n^2} \quad (\text{A7.20})$$

Step 5: Determine the quality factor Q to size of the resistor bank.

APPENDIX 7

$$R = X_n Q \quad (A7.21)$$

where: $X_n = X_{Ln} = X_{cn} = \sqrt{X_L X_C} = \sqrt{\frac{L}{C}}$ (A7.22)

Step 6: Determine the R, L and C elements of the filter.

Step 7: The impedance at any harmonic “h” is:

$$Z_F(h) = \frac{jR(h)X_L}{R + j(h)X_L} - j\frac{X_C}{h} \quad (A7.23)$$

subject to: $0.5 < Q < 5$

For example: A 34kV capacitor bank of 7.2 Mvars is to be used in the design of a 2nd – order damped filter tuned to $h_n \geq 10.7$.

Step 1: Tuning frequency $h_n \geq 10.7$

Step 2: $Q_C = 7.2$ Mvars

Step 3: $X_C = \frac{34^2}{7.2} = 160.556\Omega$

Step 4: $X_L = \frac{160.556}{10.7^2} = 1.4024\Omega$

Step 5: $X_n = \sqrt{1.4024 \times 160.556} = 15.005$

Assuming $Q = 5$, $R = 15.005 \times 5 = 75.027\Omega$

Step 6: If $f_1 = 60\text{Hz}$ $L = 1.4024/2\pi 60 = 3.7199\text{mH}$
 $C = 1/2\pi 60 \ 160.556 = 16.5212\mu\text{F}$

Therefore: $R = 75.027\Omega$, $L = 3.7199\text{mH}$, $C = 16.5212\mu\text{F}$

Subject to: $Q = 5$

Step 7: Using $Z_F(h)$, “h” can be varied over a range of frequencies to calculate the filter impedance and this can be plotted. A low impedance will be offered to $h_n \geq 10.7$ and a high impedance to $h_n < 10.7$.

Checking:

Let $h_n = 5^{\text{th}}$ harmonic, then:

$$Z_F(5^{\text{th}}) = \frac{j(75.027)(5)(1.4024)}{75.027 + j(5)(1.4024)} - j\frac{160.556}{5}$$

$$= 25.168 /_{-88.5^\circ} \Omega$$

Let $h_n = 17^{\text{th}}$ harmonic, then:

$$Z_F(17^{\text{th}}) = \frac{j(75.027)(17)(1.4024)}{75.027 + j(17)(1.4024)} - j\frac{160.556}{17}$$

$$= 14.015 /_{-60.6^\circ} \Omega$$

thus: $Z_F(5^{\text{th}}) > Z_F(17^{\text{th}})$, showing that high order harmonics are presented a lower impedance path than low order harmonics.

Note: The preferred Q for a high pass filter is 1 or 2 as this can achieve a flat response above the tuned frequency. [11]

7.3 COMPUTER MODEL INCLUDING 2ND – ORDER DAMPED FILTER AT PCC FOR CASE 3

```

TITLE TITLE1="40 kV End-User Network-Consumers 1, 2 and 3"
      TITLE2="One PF Capacitor Bank at Bus4A, Bus4B, Bus6BA and at Bus5AA"
      TITLE3="Harmonic Resonance - mpt-A - 2nd-OrderFilter at bus 3"
!
!
! Case : Harmonic Resonance case, Single-phase representation
!   of a three-phase network
!
!   Scan case
!
!   Solution will be for 1 Amp Injection
!   over a range of frequencies (60 Hz to 1200 Hz)
!
!
! -----SCAN SOURCE GROUP-----
!
SCAN NAME=SCAN1 BUS=HSOURCE1 FMIN=60 FMAX=1200 FINC=1 ANG=0.0
SCAN NAME=SCAN2 BUS=HSOURCE2 FMIN=60 FMAX=1200 FINC=1 ANG=0.0
!
!
! -----VOLTAGE SOURCE-----
!
! Utility source, Positive Sequence Source
!
VSOURCE NAME=VSRC BUS=BUS1 MAG=23094
!
!
! Positive Sequence Source Equivalent at 40 KVBUS
! Note: Impedance Value Given in Ohms at 60 Hz
!
BRANCH NAME=LINE1 FROM=BUS1 TO=BUS2 R=0.1872 X=1.1644

```

APPENDIX 7

```

!
! Transformer at SOURCE
!
TRANSFORMER  NAME = TS  MVA=20
H.1 = BUS2  X.1 = BUS3
kV.H = 40.00  kV.X =12.00
%R.HX = 1.0  %X.HX = 15.10
%Imag=0 XRCONSTANT=YES
!
! -----2ND-ORDER FILTER AT BUS3(PCC)-----
! Metering element in series with filter
!
BRANCH NAME=BRANCHA FROM=BUS3 TO=BUS4 R=0.0001 X=0.0
!
BRANCH NAME=BRANCHAA FROM=BUS4 TO=BUS4A R=0.0 X=0.31443
!
BRANCH NAME=BRANCHAB FROM=BUS4 TO=BUS4A R=3.36444 X=0.0
!
CAPACITOR NAME=CAP4A FROM=BUS4A KV=12.0 MVA=4.0
!
! -----DISTRIBUTION NETWORK 1-----
!
! 12 kV Distribution Line
! Note: Line capacitance not included due to short length
!
BRANCH NAME=LINE2 FROM=BUS3 TO=BUS5 R=0.000001 X=0.000001
!
! -----CONSUMER 1-----
!
! Transformer at entrance to Consumer 1
!
TRANSFORMER  NAME = T1  MVA=5
H.1 = BUS5  X.1 = BUS5A
kV.H = 12.00  kV.X = 6.00
%R.HX = 0.5  %X.HX = 12.0
%Imag=0 XRCONSTANT=YES
!
! LinearLoad=100% Three-phase Motor Load
!
LINEARLOAD NAME=LOAD5A FROM=BUS5A KVA=2000.0 KV=6.00 DF=0.50000
%Parallel=0.0 %Series=100
!
! PF Capacitor in Consumer 1
!
! Metering element in series with capacitor
!
BRANCH NAME=BRANCHE FROM=BUS5A TO=BUS5AA R=0.001 X=0.0
!
CAPACITOR NAME=CAP5AA FROM=BUS5AA KV=6.0 MVA=1.3244
!
! -----CONSUMER 2-----
!
! Transformer at entrance to Consumer 2
!
TRANSFORMER  NAME = T2  MVA=1.0
H.1 = BUS5  X.1 = BUS5B
kV.H = 12.00  kV.X = 0.40
%R.HX = 0.8635  %X.HX = 5.0
%Imag=0 XRCONSTANT=YES
!
! LinearLoad=100% Three-phase Motor Load
!
LINEARLOAD NAME=LOAD5B FROM=BUS5B KVA=250.0 KV=0.40 DF=0.90000
%Parallel=0.0 %Series=100
!
! -----DISTRIBUTION NETWORK 2-----
!
! 12 kV Distribution Line
! Note: Line capacitance not included due to short length

```

APPENDIX 7

```

!
BRANCH NAME=LINE3 FROM=BUS3 TO=BUS6 R=0.000001 X=0.000001
!
! -----CONSUMER 3-----
!
! Metering element for monitoring injected harmonics from drives 1 & 2
!
BRANCH NAME=LINE4 FROM=BUS6 TO=BUS6A R=0.000001 X=0.000001
!
! Transformer feeding Drive1 in Consumer 3
!
TRANSFORMER NAME = T3A MVA=5.0
H.1 = BUS6A X.1 = BUS6AA
kV.H = 12.00 kV.X = 6.0
%R.HX = 0.9885 %X.HX = 12.0
%Imag=0 XRCONSTANT=YES
!
! Metering element in series with drive1
!
BRANCH NAME=BRANCHB FROM=BUS6AA TO=HSOURCE1 R=0.0001 X=0.0
!
! Three-Phase Harmonic Source 1
!
! 2.10 MVA Drive, 6 Pulse, 6.0 kV
!
NONLINEARLOAD NAME=DRIVE1 BUS=HSOURCE1 KVA=700.00 KV=3.4641 DF=0.80
TABLE=
{
{ 1, 202.07259, 166.19}, //
{ 5, 36.3706, 110.51},
{ 7, 24.2487, 82.08},
{ 11, 12.1243, 22.63},
{ 13, 8.0829, -9.39},
{ 17, 4.0414, -82.42},
{ 19, 2.0207, -125.51} //
}
!
! Transformer feeding Drive2 in Consumer 3
!
TRANSFORMER NAME = T3B MVA=2.0
H.1 = BUS6A X.1 = BUS6AB
kV.H = 12.00 kV.X = 0.40
%R.HX = 0.9861 %X.HX = 7.0
%Imag=0 XRCONSTANT=YES
!
! Metering element in series with drive2
!
BRANCH NAME=BRANCHC FROM=BUS6AB TO=HSOURCE2 R=0.0001 X=0.0
!
! Three-Phase Harmonic Source 2
!
! 0.25 MVA Drive, 6 Pulse, 0.4 kV
!
NONLINEARLOAD NAME=DRIVE2 BUS=HSOURCE2 KVA=83.333 KV=0.23094 DF=0.80
TABLE=
{
{ 1, 360.8426, 166.19}, //
{ 5, 64.9516, 110.51},
{ 7, 43.3011, 82.08},
{ 11, 21.6505, 22.63},
{ 13, 14.4337, -9.39},
{ 17, 7.2168, -82.42},
{ 19, 3.6084, -125.51} //
}
!
! Transformer feeding pf corrected load in Consumer 3
!
TRANSFORMER NAME = T3C MVA=2.0
H.1 = BUS6 X.1 = BUS6B

```

```

kV.H = 12.00  kV.X = 0.40
%R.HX = 0.9861  %X.HX = 7.0
%Imag=0 XRCONSTANT=YES
!
! LinearLoad=100% Three-phase Motor Load in Consumer 3
!
LINEARLOAD NAME=LOAD6A FROM=BUS6B KVA=1000.0 KV=0.40 DF=0.80000
    %Parallel=0.0 %Series=100
!
! PF Capacitor in Consumer 3
!
! Metering element in series with capacitor
!
BRANCH NAME=BRANCHD FROM=BUS6B TO=BUS6BA R=0.0001 X=0.0
!
CAPACITOR NAME=CAP6BA FROM=BUS6BA KV=0.40 MVA=0.30
!
!-----RESULTS REQUIRED-----
!
RETAIN VOLTAGES=yes
RETAIN CURRENTS=yes
!
!
! End of Input File
!
....

```

7.4 VTHD% and %MI CALCULATIONS AT PCC AND BUS 5AA AFTER 2ND ORDER FILTER INSTALLED

- (s₁) VTHD% = 2.208% (< 5% IEEE 519 standard)
 $I_{C(RMS)} = 141.721A$ $1.3 I_{I(RATED)} = 165.6A$
 %MI = 85.54% (%MI < 100%)
 $V_{C1} = 3623.35V$ $V_{C1(RATED)} = 3464.1016V$
 %V_{C1} = 104.597% ($V_{C1} \leq 1.10U_N$)
- (s₂) VTHD% = 2.293% (<5% IEEE 519 standard)
 $I_{C(RMS)} = 139.458A$ $1.3 I_{I(RATED)} = 165.67A$
 %MI = 84.178% (%MI < 100%)
 $V_{C1} = 3548.72V$ $V_{C1(RATED)} = 3464.1016V$
 %V_{C1} = 102.442% ($V_{C1} \leq 1.10U_N$)
- (s₃) VTHD% = 2.397% (<5% IEEE 519 standard)
 $I_{C(RMS)} = 137.033A$ $1.3 I_{I(RATED)} = 165.67A$
 %MI = 82.71% (%MI < 100%)
 $V_{C1} = 3466.71V$ $V_{C1(RATED)} = 3464.1016V$
 %V_{C1} = 100.675V% ($V_{C1} \leq 1.10U_N$)
 dpf = 0.912 (bus 5)

APPENDIX 8

COMPUTER MODEL INCLUDING NOTCH FILTER AT BUS6B – CASE 3

8.1 COMPUTER MODEL

```
TITLE TITLE1="40 kV End-User Network-Consumers 1, 2 and 3"
      TITLE2="One PF Capacitor Bank at Bus4A, Bus4B, Bus6BA and at Bus5AA"
      TITLE3="Harmonic Resonance - mpt-A, notch FILTER at bus 6B"
!
! Case : Harmonic Resonance case, Single-phase representation
!       of a three-phase network
!
!       Scan case
!
!       Solution will be for 1 Amp Injection
!       over a range of frequencies (60 Hz to 1200 Hz)
!
!-----SCAN SOURCE GROUP-----
!
SCAN NAME=SCAN1 BUS=HSOURCE1 FMIN=60 FMAX=1200 FINC=1 ANG=0.0
SCAN NAME=SCAN2 BUS=HSOURCE2 FMIN=60 FMAX=1200 FINC=1 ANG=0.0
!
!-----VOLTAGE SOURCE-----
!
! Utility source, Positive Sequence Source
!
VSOURCE NAME=VSRC BUS=BUS1 MAG=23094
!
! Positive Sequence Source Equivalent at 40 KVBUS
! Note: Impedance Value Given in Ohms at 60 Hz
!
BRANCH NAME=LINE1 FROM=BUS1 TO=BUS2 R=0.1872 X=1.1644
!
! Transformer at SOURCE
!
TRANSFORMER NAME = TS MVA=20
H.1 = BUS2 X.1 = BUS3
kV.H = 40.00 kV.X =12.00
%R.HX = 1.0 %X.HX = 15.10
%Imag=0 XRCONSTANT=YES
!
!-----PF CORRECTION CAPACITORS AT BUS3(PCC)-----
!
BRANCH NAME=BRANCHA FROM=BUS3 TO=BUS4 R=0.0001 X=0.0
!
! PF Capacitor Bank 4A connection
!
BRANCH NAME=BRANCHAA FROM=BUS4 TO=BUS4A R=0.0001 X=0.0
!
CAPACITOR NAME=CAP4A FROM=BUS4A KV=12.0 MVA=2.0
!
! PF Capacitor Bank 4B connection
!
BRANCH NAME=BRANCHAB FROM=BUS4 TO=BUS4B R=0.0001 X=0.0
!
CAPACITOR NAME=CAP4B FROM=BUS4B KV=12.0 MVA=2.0
!
!-----DISTRIBUTION NETWORK 1-----
!
! 12 kV Distribution Line
! Note: Line capacitance not included due to short length
!
BRANCH NAME=LINE2 FROM=BUS3 TO=BUS5 R=0.000001 X=0.000001
!
```

APPENDIX 8

```

! -----CONSUMER 1-----
!
! Transformer at entrance to Consumer 1
!
TRANSFORMER  NAME = T1  MVA=5
H.1 = BUS5  X.1 = BUS5A
kV.H = 12.00  kV.X = 6.00
%R.HX = 0.5  %X.HX = 12.0
%Imag=0 XRCONSTANT=YES
!
! LinearLoad=100% Three-phase Motor Load
!
LINEARLOAD NAME=LOAD5A FROM=BUS5A KVA=2000.0 KV=6.00 DF=0.50000
%Parallel=0.0 %Series=100
!
! PF Capacitor in Consumer 1
!
! Metering element in series with capacitor
!
BRANCH NAME=BRANCHE FROM=BUS5A TO=BUS5AA R=0.0001 X=0.0
!
CAPACITOR NAME=CAP5AA FROM=BUS5AA KV=6.0 MVA=1.3244
!
! -----CONSUMER 2-----
!
! Transformer at entrance to Consumer 2
!
TRANSFORMER  NAME = T2  MVA=1.0
H.1 = BUS5  X.1 = BUS5B
kV.H = 12.00  kV.X = 0.40
%R.HX = 0.8635  %X.HX = 5.0
%Imag=0 XRCONSTANT=YES
!
! LinearLoad=100% Three-phase Motor Load
!
LINEARLOAD NAME=LOAD5B FROM=BUS5B KVA=250.0 KV=0.40 DF=0.90000
%Parallel=0.0 %Series=100
!
! -----DISTRIBUTION NETWORK 2-----
!
! 12 kV Distribution Line
! Note: Line capacitance not included due to short length
!
BRANCH NAME=LINE3 FROM=BUS3 TO=BUS6 R=0.000001 X=0.000001
!
! -----CONSUMER 3-----
!
! Metering element for monitoring injected harmonics from drives 1 & 2
!
BRANCH NAME=LINE4 FROM=BUS6 TO=BUS6A R=0.000001 X=0.000001
!
! Transformer feeding Drive1 in Consumer 3
!
TRANSFORMER  NAME = T3A  MVA=5.0
H.1 = BUS6A  X.1 = BUS6AA
kV.H = 12.00  kV.X = 6.0
%R.HX = 0.9885  %X.HX = 12.0
%Imag=0 XRCONSTANT=YES
!
! Metering element in series with drive1
!
BRANCH NAME=BRANCHB FROM=BUS6AA TO=HSOURCE1 R=0.0001 X=0.0
!
! Three-Phase Harmonic Source 1
!
! 2.10 MVA Drive, 6 Pulse, 6.0 kV
!
NONLINEARLOAD NAME=DRIVE1 BUS=HSOURCE1 KVA=700.00 KV=3.4641 DF=0.80
TABLE=

```

APPENDIX 8

```

{
{ 1, 202.07259, 166.19}, //
{ 5, 36.3706, 110.51},
{ 7, 24.2487, 82.08},
{ 11, 12.1243, 22.63},
{ 13, 8.0829, -9.39},
{ 17, 4.0414, -82.42},
{ 19, 2.0207, -125.51} //
}
!
! Transformer feeding Drive2 in Consumer 3
!
TRANSFORMER NAME = T3B MVA=2.0
H.1 = BUS6A X.1 = BUS6AB
kV.H = 12.00 kV.X = 0.40
%R.HX = 0.9861 %X.HX = 7.0
%Imag=0 XRCONSTANT=YES
!
! Metering element in series with drive2
!
BRANCH NAME=BRANCHC FROM=BUS6AB TO=HSOURCE2 R=0.0001 X=0.0
!
! Three-Phase Harmonic Source 2
!
! 0.25 MVA Drive, 6 Pulse, 0.4 kV
!
NONLINEARLOAD NAME=DRIVE2 BUS=HSOURCE2 KVA=83.333 KV=0.23094 DF=0.80
TABLE=
{
{ 1, 360.8426, 166.19}, //
{ 5, 64.9516, 110.51},
{ 7, 43.3011, 82.08},
{ 11, 21.6505, 22.63},
{ 13, 14.4337, -9.39},
{ 17, 7.2168, -82.42},
{ 19, 3.6084, -125.51} //
}
!
! Transformer feeding pf corrected load in Consumer 3
!
TRANSFORMER NAME = T3C MVA=2.0
H.1 = BUS6 X.1 = BUS6B
kV.H = 12.00 kV.X = 0.40
%R.HX = 0.9861 %X.HX = 7.0
%Imag=0 XRCONSTANT=YES
!
! LinearLoad=100% Three-phase Motor Load in Consumer 3
!
LINEARLOAD NAME=LOAD6A FROM=BUS6B KVA=1000.0 KV=0.40 DF=0.80000
%Parallel=0.0 %Series=100
!
! Notch Filter in Consumer 3
!
BRANCH NAME=BRANCHD FROM=BUS6B TO=BUS6BA R=0.22694 X=0.024143
!
CAPACITOR NAME=CAP6BA FROM=BUS6BA KV=0.40 MVA=0.30
!
!-----RESULTS REQUIRED-----
!
RETAIN VOLTAGEs=yes
RETAIN CURRENTS=yes
!
! End of Input File
!
....

```

8.2 VTHD% AND %MI CALCULATIONS – NOTCH FILTER – SOLUTION 2Bus 5AA (end-user 1)(s₁) VTHD% = 4.087% (< 5% IEEE 519 standard)

$$I_{C(RMS)} = 154.389A \quad 1, 3 I_{I(RATED)} = 165.67A$$

$$\%MI = 93.19\% \quad (\%MI < 100\%)$$

$$V_{C1} = 3620.62V \quad V_{C1(RATED)} = 3464.1016V$$

$$\%V_{C1} = 104.518\% (V_{C1} \leq 1.10U_N)$$

(s₂) VTHD% = 4.0116% (<5% IEEE 519 standard)

$$I_{C(RMS)} = 152.055A \quad 1.3 I_{I(RATED)} = 165.67A$$

$$\%MI = 91.781\% \quad (\%MI < 100\%)$$

$$V_{C1} = 3546.06V \quad V_{C1(RATED)} = 3464.1016V$$

$$\%V_{C1} = 102.365\% (V_{C1} \leq 1.10U_N)$$

(s₃) VTHD% = 3.9668% (<5% IEEE 519 standard)

$$I_{C(RMS)} = 149.631A \quad 1.3 I_{I(RATED)} = 165.67A$$

$$\%MI = 90.318\% \quad (\%MI < 100\%)$$

$$V_{C1} = 3464.12V \quad V_{C1(RATED)} = 3464.1016V$$

$$\%V_{C1} = 100.00V\% (V_{C1} \leq 1.10U_N)$$

$$dpf = 0.912 \text{ (bus 5)}$$

8.3 VTHD% AND %MI CALCULATIONS – NOTCH FILTER – SOLUTION 3Bus 5AA (end-user 1)(s₁) VTHD% = 4.9641% (< 5% IEEE 519 standard)

$$I_{C(RMS)} = 142.016A \quad 1, 3 I_{I(RATED)} = 165.67A$$

$$\%MI = 85.72\% \quad (\%MI < 100\%)$$

$$V_{C1} = 3784.54V \quad V_{C1(RATED)} = 3464.1016V$$

$$\%V_{C1} = 109.249\% (V_{C1} \leq 1.10U_N)$$

(s₂) VTHD% = 4.97147% (<5% IEEE 519 standard)

$$I_{C(RMS)} = 139.137A \quad 1.3 I_{I(RATED)} = 165.67A$$

$$\%MI = 83.984\% \quad (\%MI < 100\%)$$

$$V_{C1} = 3706.33V \quad V_{C1(RATED)} = 3464.1016V$$

$$\%V_{C1} = 106.992\% (V_{C1} \leq 1.10U_N)$$

(s₃) VTHD% = 4.9796% (<5% IEEE 519 standard)

$$I_{C(RMS)} = 135.973A \quad 1.3 I_{I(RATED)} = 165.67A$$

$$\%MI = 82.07\% \quad (\%MI < 100\%)$$

$$V_{C1} = 3620.4V \quad V_{C1(RATED)} = 3464.1016V$$

$$\%V_{C1} = 104.51V\% (V_{C1} \leq 1.10U_N)$$

$$dpf = 0.95 \text{ (bus 5)}$$

APPENDIX 9

EXAMPLES – DECISION ANALYSIS

9.1 DECISION PROBLEM WHEN A SITUATION OF CERTAINTY EXISTS

A problem from reference [24] is chosen to explain how to develop a quantitative decision model when the situation is one of certainty. *The following worked example is my approach on how a decision model is constructed for investigating a decision problem when a situation of certainty exists:*

DECISION PROBLEM

- a. A vendor buys apples each day and sells them on the streets.
- b. He buys them at so much per kilogram (cents/kilogram).
- c. He can choose a different selling price each day. Once he has chosen he keeps the same price for his whole working day.
- d. At the end of each working day, all unsold apples must be sold for a salvage of so much per kilogram (cent/kilogram).
- e. The vendor must “decide”:
 - (1) How many kilograms of apples to buy per day, and
 - (2) What price to charge for them.
- f. He wants to earn the highest possible profit per day.

DECISION MODEL

Keeping the general structure for a decision model in mind we proceed as follows:

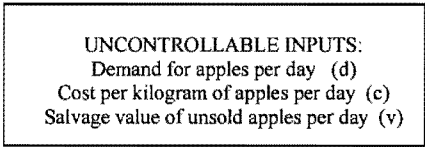
Step 1 Choose the controllable inputs, that is, the variables under the control of the decision maker (vendor).

Let, b = number of kilograms of apples the vendor will buy per day.
 s = selling price per kilogram of apples.

CONTROLLABLE INPUTS:
Number of apples the
vendor will buy per day (b)
selling price per day (s)

Step 2 Choose the uncontrollable inputs, that is, the variables not under the control of the decision maker.

Let, d = demand (in kilograms) for apples per day.
 c = cost per kilogram (cents/kilogram) of apples per day
 v = salvage value of unsold apples (cents/kilogram) per day.



Step 3 Choose the payoff measure (outcome), that is, the objective.

Let P = profit (\$) per day (A9.1)

This completes the definition of the variables.

Step 4 Mathematically relate the payoff measure (objective) to the controllable and uncontrollable inputs by means of an equation called the objective function for the decision problem and take into account constraints.

Let x = sales in kilograms per day.

The objective function (OBF) may be formulated as follows:

OBF = $P = \begin{cases} x(s - c) & \text{(if } d \geq b) \\ x(s - c) - (b - x)(c - v) & \text{(if } d < b) \end{cases}$ (A9.2)

stated verbally,

- a. If the demand (d) is greater than or equal to the quantity purchased (b), sales will equal demand and profit (P) will equal sales (x) times profit per unit, where profit per unit is selling price minus cost (s – c).
- b. If demand (d) is less than the quantity purchased (b), there is a profit (P) of (s – c) on sold units (x) and a loss of (c – v) on the unsold units (b – x).

Step 5 Construct the decision model.

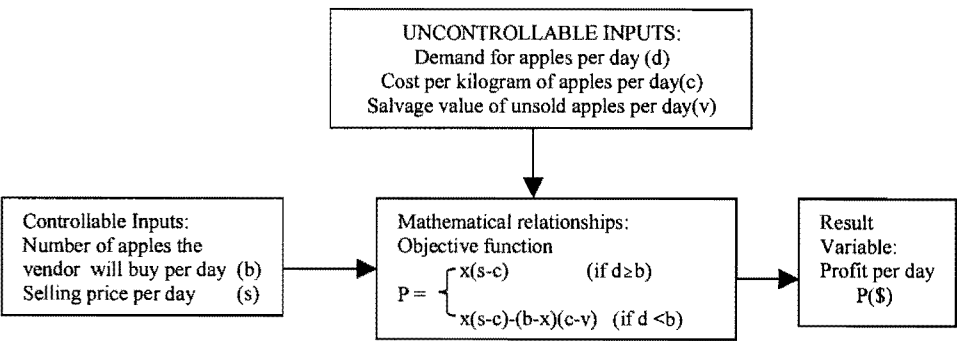


Figure A9.1 Decision Model

Step 6 Establish the situation for the uncontrollable inputs.

A model requires data on the values of the uncontrollable inputs.

The vendor buys apples each day at a cost of 30 cents/kg (c). All unsold apples are sold at the end of each day for a salvage value of 20 cents/kg (v).

Thus (c) and (v) are known “for sure”, that is, these aspects of the decision problem have a situation known as “certainty”. These uncontrollable inputs are therefore under the control of the decision maker, thus the number of controllable inputs increases:

Controllable inputs
(b), (s), (c) and (v)

Sales (x) is a partially controllable variable as it is affected by the amount purchased (b) and therefore available for sale (b, controllable) and by the demand (d, uncontrollable).

Now relating sales (x) to demand (d) and the quantity bought (b),

$$x = \begin{cases} b & \text{(if } d \geq b) \\ d & \text{(if } d < b) \end{cases} \quad (\text{A9.3})$$

- a. If (d) is greater than (b), the vendor sells out ($x = b$).
- b. If (d) is less than (b), ($x = d$).

The vendor knows for “certain” (from experience) that (s) is related to (d) by the following graph:

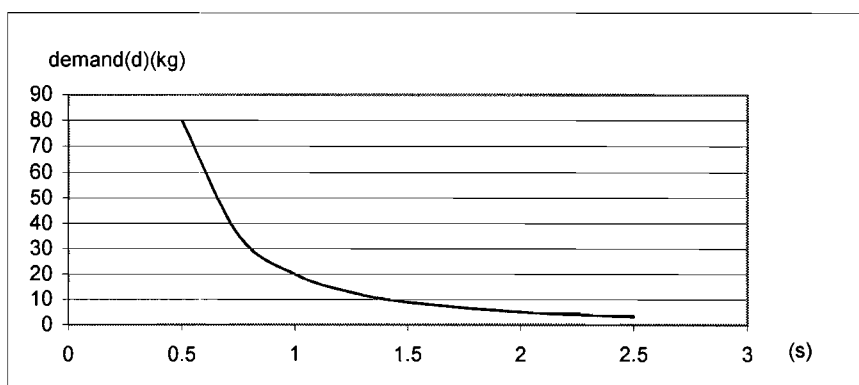


Figure A9.2 Demand versus selling price

APPENDIX 9

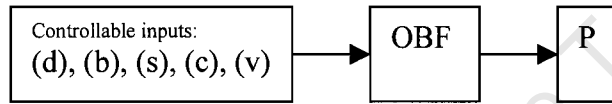
Which stated mathematically is:

$$d = \frac{20}{s^2} \quad (\text{A9.4})$$

subject to: (for all $s \geq 0$)

Therefore, if (s) is chosen, the decision maker knows for “certain” the value of (d). If he knows (d) with certainty, then he knows the quantity (b) to be purchased with certainty ($b = d$).

If you know (d) with certainty, then you know (x) for certain ($d = x$). Thus (d) is no longer uncontrollable but becomes controllable (d, controllable), therefore,



In general, if an alternative is dominated by another alternative, it can be eliminated from the possible choices [25].

The OBF consists now of only controllable inputs. As $b = d$, (b) can be removed as a controllable input. Likewise as (d) depends on (s), (d) can also be removed. As, ($b = d = x$), there is no need to consider (v) and it can also be removed as a controllable input.

As ($b = d$), the OBF of equation (A9.2), reduces to:

$$P = x(s - c) \quad (\text{A9.5})$$

$$\text{Subject to: } (b = d) \quad (\text{A9.6})$$

Further, as ($d = x$), (x) in the OBF can be replaced by $20/s^2$, thus,

$$P = \frac{20}{s^2}(s - c) \quad (\text{A9.7})$$

$$\text{subject to: } (b = d) \quad (\text{A9.8})$$

$$P = \frac{20}{s} - \frac{20c}{s^2} \quad (\text{A9.9})$$

$$\text{subject to: } (b = d) \quad (\text{A.10})$$

APPENDIX 9

As ($c = \$0.3$), and is known for sure, (c) can be removed as a controllable input and the OBF, becomes,

$$\text{OBF} = P = \frac{20}{s} - \frac{6}{s^2} \quad (\text{A9.11})$$

$$\text{subject to : } s \geq c \quad (\text{A9.12})$$

$$s \geq 0 \quad (\text{A9.13})$$

$$b = d = 20/s^2 \quad (\text{A9.14})$$

The model is simplified to only one decision alternative:

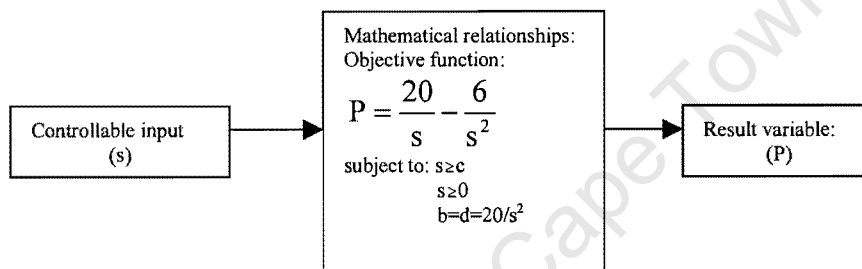


Figure A9.3 Decision Model – Situation of certainty

Step 7 Find the best choice by determining the effect that the controllable input has with respect to the objective taking constraints into account (generalized framework – step 3). As the decision maker wishes to earn the highest possible profit (P) per day, he seeks the optimal decision (s), namely:

$$\text{Maximize } (s) \left\{ P = \frac{20}{s} - \frac{6}{s^2} \right\} \quad (\text{A9.15})$$

$$\text{subject to: } s \geq c \quad (\text{A9.16})$$

$$s \geq 0 \quad (\text{A9.17})$$

$$b = 20/s^2 \quad (\text{A9.18})$$

One can establish the optional value by trial and error or by calculus. To derive the optimal selling price (s^*), optimal purchase quantity (b^*) and the optimal profit (P^*) by calculus, we proceed as follows:

Generalizing equation A9.4, assume (d) = k/s^2 , then from equation (A9.7),

APPENDIX 9

$$P = \frac{k}{s} - \frac{kc}{s^2} \quad (\text{A9.19})$$

Taking the first derivative of (P) with respect to (s) and setting the result equal to zero, we obtain,

$$\frac{dP}{ds} = -\frac{k}{s^2} + \frac{2kc}{s^3} = 0 \quad (\text{A9.20})$$

$$s = 2c \quad (\text{A9.21})$$

Taking the second derivative to distinguish between a maximum or minimum value, we obtain,

$$\frac{d^2P}{ds^2} = \frac{2k}{s^3} - \frac{6kc}{s^4} = \frac{2ks - 6kc}{s^4} = \frac{2k(s - 3c)}{s^4} \quad (\text{A9.22})$$

for $s = 2c$,

$$\frac{d^2P}{ds^2} = \frac{2k(2c - 3c)}{s^4} = \frac{2k(-c)}{s^4} = \frac{-2kc}{s^4} \quad (\text{A9.23})$$

and so the value $s = 2c$ is a maximum value, therefore:

$$s^* = 2c \quad (\text{A9.24})$$

$$b^* = \frac{k}{s^2} = \frac{k}{(2c)^2} = \frac{k}{4c^2} \quad (\text{A9.25})$$

$$P^* = \frac{k}{2c} - \frac{kc}{4c^2} = \frac{2ck - kc}{4c^2} = \frac{kc}{4c^2} = \frac{k}{4c} \quad (\text{A9.26})$$

Now as, $c = 30c$ (\$0.30)

$$P^* = \frac{20}{(4)(0.3)} = \frac{5}{0.3} = \$16.67 \quad (\text{A9.27})$$

SOLUTION TO DECISION PROBLEM

Thus, the vendor must price his apples at 60c/kg ($s = 2c$) and he should buy 55.56kg of apples per day ($b^* = 20/(4)(0.3)^2 = 55.56\text{kg}$) and he will sell 55.56kg per day and earn a daily profit (P^*) of \$16.67.

Following on with the example, the problem as applicable to the decision theory process:

PROBLEM STATEMENT

The vendor establishes that he can only order apples in boxes of 100kg and his maximum storage capacity is 400kg. From past experience he knows that daily demand is in the range of 0 to 400kg in boxes of 100kg. He has decided to sell his product at 50 cent/kg. As the cost/kg = 30 cents his revenue is 20 cents/kg. On each kilogram not sold the loss is 10 cents/kg as the salvage value (v) is 20 cents/kg. His objective is to earn the highest profit (P).

For this problem:

- a. There are 5 decision alternatives (0, 100, 200, 300, 400).
- b. There are 5 possible values of daily demand (0, 100, 200, 300, 400). He knows the demand is for 100kg boxes of apples.
- c. The objective function for the problem is:

$$P = \begin{cases} x(s - c) & (d \geq b) \\ x(s - c) - (b - x)(c - v) & (d < b) \end{cases} \quad (A9.28)$$

using, the objective function, the outcomes can be determined:

1. If 0kg is bought, sales (x) is zero and revenue = \$0, irrespective of demand.
2. If 100kg is bought

When d = 0, (d < b)

$$\begin{aligned} P &= x(s - c) - (b - x)(c - v) & (d < b) & \quad (A9.29) \\ &= 0(0.5 - 0.3) - (100 - 0)(0.3 - 0.2) \\ &= -\$10 \end{aligned}$$

When d = 100, (d = b)

$$\begin{aligned} P &= x(s - c) & (d \geq b) & \quad (A9.30) \\ &= 100(0.2) = \$20 \end{aligned}$$

When d = 200, 300 or 400, (d > b)

P = \$20 in each case

3. If 200kg is bought

When d = 0, (d < b)

APPENDIX 9

$$P = 0(0.2) - (200 - 0)(0.1) = -\$20$$

When $d = 100$, ($d < b$)

$$P = 100(0.2) - (200 - 100)(0.1) = \$10$$

When $d = 200, 300$ or 400 , ($d \geq b$)

$$P = 200(0.2) = \$40 \text{ in each case}$$

4. If 300kg is bought

When $d = 0$ ($d < b$)

$$P = 0(0.2) - (300 - 100)(0.1) = -\$30$$

When $d = 100$, ($d < b$)

$$P = 100(0.2) - (300 - 100)(0.1) = \$0$$

When $d = 200$, ($d < b$)

$$P = 200(0.2) - (300 - 200)(0.1) = \$30$$

When $d = 300$ or 400 , ($d \geq b$)

$$P = 300(0.2) = \$60 \text{ in each case}$$

5. If 400kg is bought

Demand versus profit					
Demand (kg)	0	100	200	300	400
Profit (\$)	-40	-10	20	50	80

Table A9.1 Demand versus profit

- d. Formulate the decision table

For any feasible pair of decision elements, the outcome is defined to be the objective function value.

Table A9.2 presents the decision model in the form of a decision (payoff) table:

DECISION TABLE					
DECISION ALTERNATIVES (a_1, a_2, \dots, a_n)	STATES OF NATURE				
	DEMAND (s_1, s_2, \dots, s_N)				
	$s_1 = 0$	$s_2 = 100$	$s_3 = 200$	$s_4 = 300$	$s_5 = 400$
a_1 : buy 0	0	0	0	0	0
a_2 : buy 100	-10	20	20	20	20
a_3 : buy 200	-20	10	40	40	40
a_4 : buy 300	-30	0	30	60	60
a_5 : buy 400	-40	-10	20	50	80

Table A9.2 Decision Table

A decision under certainty occurs when you know which state of nature will happen. Alternatively, it can be seen as a case (decision table) with only a single state of nature (one column).

Let us continue with the vendor scenario. Lets assume that the vendor does not have to buy apples until the end of each day and then only after he has taken orders for that day. His arrangement with his clients is to deliver the ordered apples after the close of each working day.

The demand is known with complete certainty.

Table A9.3, is the decision table relevant to this decision problem. If the demand is 100kg, then the decision table is:

DECISION TABLE	
DECISION ALTERNATIVES (a_1, a_2, \dots, a_n)	STATE OF NATURE
	DEMAND
	($s_2 = 100$)
a_1 : buy 0	0
a_2 : buy 100	20
a_3 : buy 200	10
a_4 : buy 300	0
a_5 : buy 400	-10

Table A9.3 Decision Table: Certainty situation

The outcomes for a demand of 100kg are: 0, 20, 10, 0, - 10.

The optimal decision is to choose the decision alternative that yields the best value (highest profit).

His decision is thus to select $a_2 = 100\text{kg}$ (buy 100kg) as it gives the maximum profit of \$20.

9.2 DECISION PROBLEM WHEN OUTCOMES ARE NON-MONETARY VALUES – ASSIGNMENT OF UTILITY VALUES

A lecturer in charge of a remedial reading program has to choose one of two books to be used in his course. The effectiveness of the books is dependant upon the attitude of the pupils toward the program. He has many years of experience and estimates that 30% (p

APPENDIX 9

=0.3) of the time the pupils are interested, 50% ($p = 0.5$) of the time they are indifferent and 20% ($p = 0.2$) of the time they are disinterested. The overall percentage improvement in reading scores and which are the outcomes for the problem are given in the decision table for this problem [24].

DECISION TABLE : REMEDIAL READING PROGRAM			
DECISION ALTERNATIVES	STATES OF NATURE		
	Interested	Indifferent	Disinterested
Book 1	100%	30%	0%
Book 2	30%	60%	30%
PROBABILITIES	0.3	0.5	0.2

Table A9.4 Decision Table – Remedial reading program

The outcomes in the table are non-monetary values (not payoff's). Therefore the EMV rule cannot be applied, as it is improper to calculate expected values of percentages.

We now need to find utility values for the outcomes in the decision table. Firstly, the outcomes are ranked from best to worst.

Ranked Outcomes

100% increase (best)
60% increase
30% increase
0% increase (worst)

Next, decide on the scale to be used. The scale 0 to 1 is used. Now assign utilities to the best and worst outcomes. Let 0 be the value for 0% increase and 1 the value for 100% increase.

We next need to find utility values for the in-between outcomes by applying the variable probability method. For $U(60\%)$, the decision-maker is asked to visualize the following hypothetical situation.

- Alternative 1. Obtain a 60% increase as a sure outcome
- Alternative 2. Obtain a 100% increase (best outcome) with a probability of p or obtain a 0% increase with probability $(1-p)$

In the interview, the decision-maker is asked a series of these questions until the p_{ind} value is determined. Trial and error questioning is usually required to identify a decision-makers p_{ind} value. When beginning the questioning, an arbitrary probability value is offered, say 0.85. If the answer is, alternative 1 is preferred, then the situation is made progressively more attractive by slowly increasing p , until p_{ind} is found. Alternatively, if the preference is option 2 initially, then p should be adjusted downward to the indifference point by a similar trial and error process.

APPENDIX 9

After questioning, let us assume for the purpose of this example that the p_{ind} value for this in-between outcome is 0.9. The utility value is as follows (chapter 3, equation 3.18):

$$U(60\%) = (0.9)U(100\%) + (0.1)U(0\%)$$

but as,

$$U(100\%) = 1 \text{ and } U(0\%) = 0,$$

$$U(60\%) = (0.9)(1.0) + (0.1)(0.0) = 0.9$$

The utility value for the non-monetary outcome of 60% increase is therefore 0.9.

In a similar manner, we can find the utility value for the non-monetary outcome of 30% increase. Again, for this example, let its p_{ind} value be 0.7. The utility is therefore 0.7.

The utility table for this decision problem is:

UTILITY TABLE: REMEDIAL READING PROGRAM			
DECISION ALTERNATIVES	STATES OF NATURE		
	Interested	Indifferent	Disinterested
Book 1	1	0.7	0
Book 2	0.7	0.9	0.7
PROBABILITIES	0.3	0.5	0.2

Table A9.5 Utility Table – Remedial reading program

To assist with the decision-making, that is, choose which book to use, we apply the expected utility value (chapter 3, equation 3.13), namely:

$$EU_{(book1)} = (0.3)(1.0) + (0.5)(0.7) + (0.2)(0) = 0.65$$

$$EU_{(book2)} = (0.3)(0.7) + (0.5)(0.9) + (0.2)(0.7) = 0.8 \text{ (largest)}$$

On the basis of the largest EU value, the decision-maker should use book 2.